USING TRANSFORMATIVE TECHNOLOGY TO TEACH CENTRAL TENDENCY AND PROMOTE EQUITY

S. Asli Özgün-Koca, Jennifer M. Lewis, Christopher Nazelli, and Lenuel Hernandez
Wayne State University
Detroit, Michigan, USA
aokoca@wayne.edu

This paper documents the design and enactment of a lesson using technology to advance ambitious and equitable statistics learning in a high school mathematics class for students of color living in poverty. The lesson was appraised using Equity-centered Transformative Technology (EQTTech) Lesson Analysis Tool, with special emphasis on the development of authority, identity, and agency. Our analysis shows how technology can support the development of positive mathematical identity, a sense of authority, and the growth of student agency in statistics instruction.

INTRODUCTION

In his April 2024 Presidential message to the National Council of Teachers Mathematics, Kevin Dykema (2024) urged mathematics teachers to "develop deeper data literacy skills with our students. Data literacy is a critical global citizenship skill that requires a strong conceptual understanding of statistics" (Paragraph 6). Data literacy, statistical literacy, statistical thinking, and statistical reasoning have been defined in different ways across the literature. Ben-Zvi and Garfield (2004) defined statistical literacy as basic and important skills, such as organizing data and creating representations, and statistical reasoning as making sense of statistical information. Statistical thinking "involves an understanding of why and how statistical investigations are conducted and the 'big ideas' that underlie statistical investigations" (Ben-Zvi & Garfield, 2004, p. 7). These definitions suggest sense-making needs to be at the center of statistics instruction. Students make sense when they are "making interpretations based on sets of data, representations of data, or statistical summaries of data…connecting one concept to another (e.g., center and spread)" (Ben-Zvi & Garfield, 2004, p. 7). Creating a teaching and learning environment for statistics with sense-making at the center requires teachers to support students' reasoning, problem-solving, and communication skills.

In this paper, we explore the implementation of a statistics lesson designed to promote equity in a high school mathematics classroom. We understand equitable teaching to be "an intentional act that teachers pursue through teaching practices that help them to view each and every student as capable of doing mathematics" (Berry, 2019, Paragraph 2). Further, the lesson uses technology to support teaching for equity, especially technologies that support sense-making, because sense-making is key to understanding. Our objective was to see whether and how various technologies might contribute to an equitable teaching and learning environment for statistics instruction.

In 2020, the National Council of Teachers of Mathematics and the American Statistical Association released *Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II: A Framework for Statistics and Data Science Education* (GAISE II) with this unequivocal message, "The ultimate goal: Statistical literacy for all" (p. 5). The GAISE II document shared a framework for statistical problem-solving processes: (i) Formulate Statistical Investigative Questions, (ii) Collect/Consider the Data, (iii) Analyze the Data, and (iv) Interpret the Results. Analyzing data is also listed as one of the key elements for reasoning and sense-making with statistics and probability in *Focus in High School Mathematics: Reasoning and Sense Making*, which claimed students gain "insight about a solution to a statistics research question by collecting data and describing features of the data using graphical and tabular representations and numerical summaries" (Shaughnessy, Chance, & Kranendonk, 2009, p. 2).

GAISE II noted in its *Analyze the Data* section that students need to "learn to use the key features of distributions for quantitative variables, such as: center: mean as the balance point, and median as the middle-ordered value, variability: interquartile range and mean absolute deviation (MAD), shape: symmetric or asymmetric and number of modes" (p. 18). As one of the key components of statistical distributions, we selected measures of central tendency as the main topic of our lesson, highlighting mean as the balance point and median as the middle-ordered value. Konold and Higgins (2003) stated

In: Kaplan, J. & Luebke, K. (Ed.) (2024). *Connecting data and people for inclusive statistics and data science education. Proceedings of the Roundtable conference of the International Association for Statistics Education (IASE), July 2024, Auckland, New Zealand.*

> Research involving students from middle school through college suggests that few students know much more about means than how to compute them…even children who can compute the mean often do not understand why the computation works or what the result represents. Although the add-and-divide algorithm is relatively simple to execute, developing a conceptual underpinning that allows one to use the mean sensibly is surprisingly difficult (p. 204).

This was echoed by Shaughnessy (2007) who highlighted the need for unpacking students' understanding of this important statistical concept: "Indeed the mean is a very powerful idea in statistics, and it is fundamental for students' understanding of summary statistics and statistical tests" (p. 965). Mokros and Russell (1995) conducted interviews with students and uncovered five approaches students use when solving average problems: average as mode, average as algorithm, average as reasonable, average as midpoint, and average as mathematical point of balance. As GAISE II suggested, understanding the mean as balance point requires intentional instruction. Some researchers have suggested tasks including interactive virtual and physical manipulatives like connecting cubes, Cuisenaire rod trains, balance beams or software to support students' thinking for average as fair-share value or average as balance point (Daiga & Driskell, 2021; Peters et al., 2016).

Our lesson design using technology grew from the understanding that "modern statistical practice is intertwined with technology; thus, it is recommended that technology be embraced to the greatest extent possible within a given circumstance" (GAISE II, 2002, p. 11). Chance et al. (2007) noted "it is hard to imagine teaching statistics today without using some form of technology" (p. 1) and they highlighted the ways technology can support students' statistical thinking, for example, by automation of calculations, emphasis on data exploration with easy access to multiple displays of data, and visualization of abstract statistical concepts and processes. Lane-Getaz (2006) enumerated ways to support students' statistical thinking. The first and crucial step is to choose and use software for teaching statistics. Sedlmeier (2014) warned that "a special emphasis should be given to a systematic and sound evaluation of such technological tools" (p. 1) grounded in solid technological, pedagogical and content-focused theoretical bases. In this lesson, we chose to use two technologies, GeoGebra and Padlet. In the next section we describe the technological and pedagogical considerations involved in their selection.

LESSON SUMMARY

A lesson was designed and conducted in two ninth-grade classes at a local US high school where 99.6% of students were Black and 75% of students qualified for free and reduced lunch. Past achievement data showed approximately 8% of students in the school were proficient in mathematics. The authors designed and delivered this lesson as a supplement to the mandated curriculum. Basic statistical concepts such as the measures of central tendency had already been covered. This lesson was adapted from an NCTM Illuminations task using an applet created specifically for the task (https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Mean-and-Median/). Since the NCTM Illuminations Interactive applet was no longer available, we substituted the GeoGebra applet, *Measures of Center-Dot Plot*, developed by the Education Development Center (EDC, https://www.edc.org/ and https://www.geogebra.org/u/edc+in+maine#people). Initially designed for geometry and algebra, GeoGebra software has evolved over the years to include statistical calculations and displays. In this lesson, the applet created in GeoGebra mainly uses graphing and basic descriptive statistical calculations and it allows students to create data sets by dragging X symbols on the left-hand side of the screen onto a dot plot graph window and then those X symbols become data points. If the "Show measures of center" box is checked, the applet calculates the mean and median of the data set and places arrows to display the measures on the plot (see Figure 1). Students can then move the data points on the graph and make conjectures about how movement of data points affects central tendency measures. We also used Padlet, a shared digital board, for students to make their mathematical thinking public and share the graphs that they created.

The lesson was comprised of four components: an introduction, a student exploration with the GeoGebra applet, a whole-class discussion incorporating postings on Padlet, and a brief individual assessment at the end of the lesson. To provide a real-world context, a scenario was shared with the students in the introduction: two employees discussing whether their salaries were typical for other employees in their company. This was followed by a quick review of measures of central tendency.

Students were asked rhetorically how measures of central tendency might add to the interpretation of salary data such as those in the scenario above.

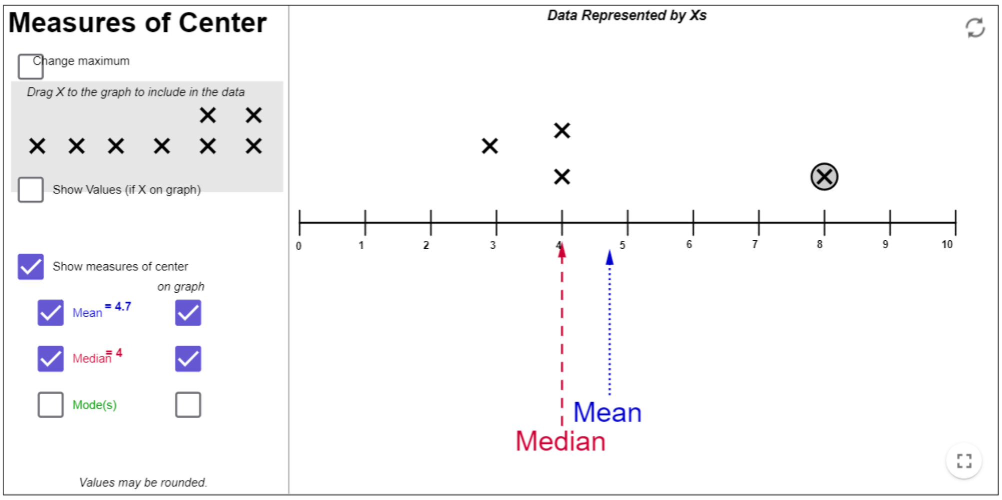


Figure 1. The GeoGebra Applet (https://www.GeoGebra.org/m/zrXEBKss)

The GeoGebra applet was then introduced to students as a tool for visualizing and exploring measures of central tendency. Dick and Hollebrands (2011) categorized GeoGebra applets as mathematical action technologies that allow students to act on mathematical objects and create meaning for themselves. The first task in GeoGebra prompted students to create dot plots of two unique data sets of ten data points between 0 and 10, each data set with mean and median of 5. Once they created their data sets, students were asked to take a screenshot of their dot plots and post it to an online sharing platform, Padlet. Padlet was used as a conveyance technology (Dick & Hollebrands, 2011), through which student responses could be gathered, shared publicly, and examined. A discussion followed about why dot plots looked different yet had the same mean and median of 5. The second task asked students to create a data set with ten data points for which the median seemed like a typical point in the data set (being closer to most of the data points) and the mean not. Students posted screenshots of these plots to a new Padlet. This was followed by a class discussion about how the different arrangements of data values affected the mean and median. The GeoGebra applet and Padlet were accessible via laptop computers or their personal devices.

Finally, students were given a data set of ten salaries in a fictional local business, where the management earned significantly more than the other workers. This was a summative assessment and brought the real-world connection back into the lesson. Students were asked to create different messages about what workers earn at the company, using the central tendency measures of median and mean. We chose to keep the applet task context-free so that students could focus solely on the placement of data points to affect the mean and median, with context incorporated in the introduction to the lesson and the assessment at the end, as described above.

LESSON ANALYSIS

To analyze the advancement of equity in the design and enactment of this lesson, we used Suh's Equity-centered Transformative Technology (EQTTech) Lesson Analysis Tool (Suh, 2016; Suh et al. 2022). Table 1 shows the analysis of lesson design and enactment along the five dimensions of the EQTTech tool:

- Dimension 1: Access to Inquiry-based Learning
- Dimension 2: Math Identity through Authorship and Agency
- Dimension 3: Formative Assessment & Differentiation
- Dimension 4: Empowerment Through Collective Thinking
- Dimension 5: Application of Mathematical and Cognitive Processes

All five dimensions of the EQTtech tool were used to analyze dimensions of student identity, authority, and agency. As discussed above, statistical reasoning and thinking demand conceptual understanding of big ideas, through which students make sense of and connect key statistical concepts. In the next paragraphs we share how student identity, authority, and agency were key to creating an environment to support students' statistical thinking.

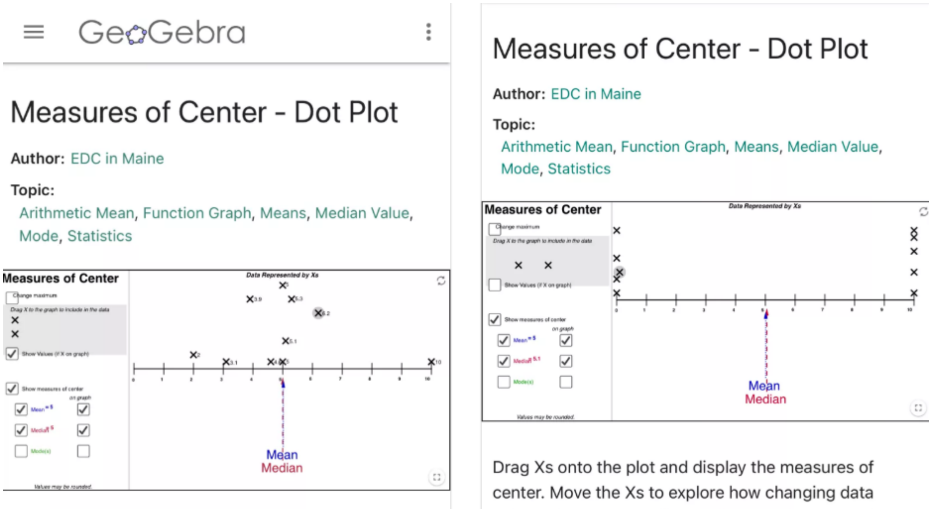
Table 1. Analysis of Lesson Design and Enactment

EQTTech Dimensions	**Lesson Design**	**Lesson Enactment**
Dimension 1: Access to Inquiry-based Learning	The applet-supported task provided access to inquiry-based learning by allowing students to interact with the technology and make conjectures.	**Identity Building:** Students received private, individualized, nonjudgmental feedback from the applet, functioning as knowledgeable or "expert other" (Edwards, 2001, p 50).
Dimension 2: Math Identity through Authorship and Agency	The task allowed students to create data sets, rather than assigning calculations on a prescribed data set. Student work was shared on a Padlet making students' individual thinking public.	**Agency Building:** The task and interaction with technology created student willingness to engage as a doer of mathematics. **Authority Building:** Students determined who made mathematical contributions, and their legitimacy. Classroom discussion allowed students to come to shared understandings.
Dimension 3: Formative Assessment & Differentiation	Individualized summative assessment connected the work on GeoGebra and a Padlet to a real-life problem.	**Agency Building:** Students were asked to convey a message about a company using statistical measures.
Dimension 4: Empowerment Through Collective Thinking	Sharing work on a Padlet allowed for collective thinking.	**Authority Building:** Students had a choice to share their work and communicate their thinking with others anonymously or using their names. Many returned to the Padlet to add their names. **Agency Building:** This lesson anchored the instruction and discussion in students' understandings and what they chose to share on Padlet.
Dimension 5: Application of Mathematical and Cognitive Processes	The task prompted students to think about alternative data sets and to come up with more than one solution that would result in the same mean and median and potentially see the mean as balance point.	**Identity Building:** Students had access to engaging content in a worthwhile task and choice to communicate their thinking with others, positioning students as owners of ideas worth communicating.

Table 1 demonstrates how the development of identity, authority, and agency may have been amplified in the enactment of the lesson. During the whole-class discussion, when the teacher asked the students where to put a data point on the graph to make the mean as 5, the following exchange transpired between student and teacher:

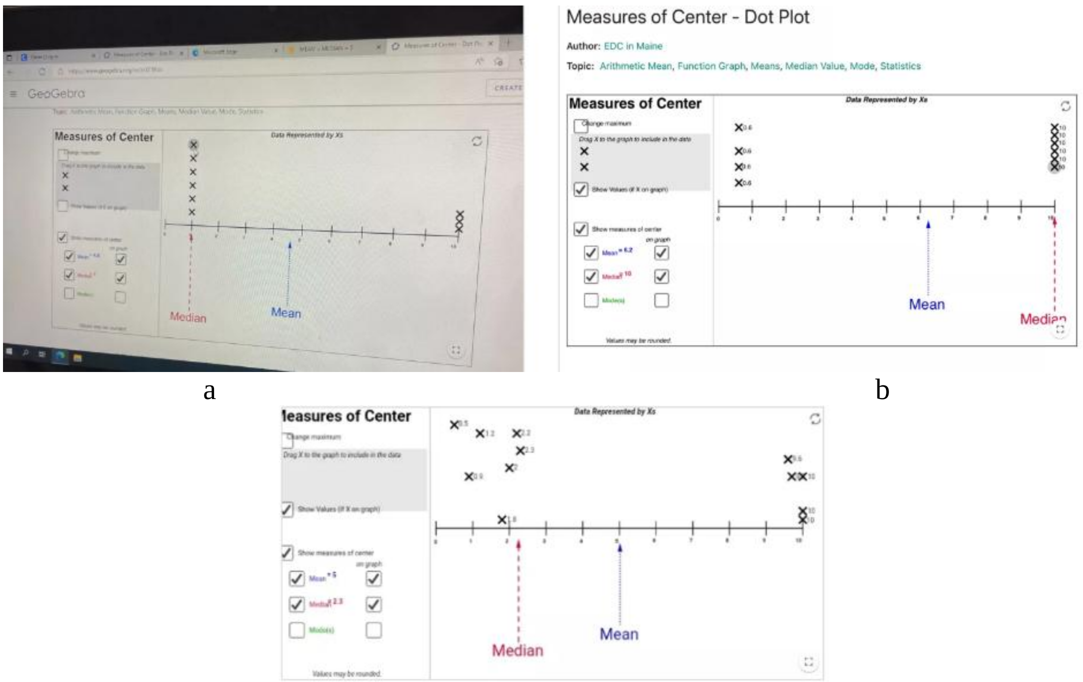
> Student: You do not need to put [the data point] directly on 5.
> The teacher then projected GeoGebra showing 5 or 6 points on the graph, and asked, "What would I have to do so my mean is 4.8 and my median is 5.1?" [meaning where to put additional data point on the graph]
> Student: Above.

In this fleeting exchange, we see how technology invited student expression of authority. The student engaged with technology and made conjectures. As a result of those conjectures, the student was able to share his answer as the owner of mathematical ideas. First, the student pointed out that the data point did not need to be at 5 (there were graphs created and shared where all data points were at 5 resulting with a mean and median of 5) and there were other arrangements of data possible as a result of his reasoning. Moreover, the student conjectured that plotting a point a little above 5 would pull the mean towards 5, hence in this instance, perhaps the idea of mean as balance point is beginning to develop. It is possible that this student's previous interactions with the applet had led him to guess that a data point higher than 5 would shift the mean from 4.8 to 5. Furthermore, the concept of mean as balance point was more accessible on the Padlet, where student work products were displayed next to one another. Students needed to interpret two different images of mean as balance point when comparing the two graphs in Figure 2. In Figure 2a, multiple data points on the left-hand side of the graph are balanced by a single data point on the far right-hand side of the graph. In Figure 2b, two nearly equivalent groups of data points are clustered equidistant from the mean. The contrasting displays allowed students to reflect on the center and spread of the two data sets.



a b

Figure 2. Padlet Screenshot for Task 1 (mean and median of 5)



a b

c

Figure 3. Padlet screenshot for Task 2

A second task asked students to create a data set with ten data points where the median was closer to more of the data points than the mean (Figure 3). Students came up with a variety of data sets for this task. Again, interaction with the applet was followed by sharing work on a shared Padlet so that students could "see" mean as balance point and median as the middle-ordered value. Explaining how she separated mean and median, one student said,

> I put a certain amount on this side [pointing to the right-hand side of a graph similar to Figure 3c] to make it 10 or equal to 10, and then I guess we were supposed to not make the mean and the median not this thing [putting her hands together], so basically I put a certain amount on this side [pointing to the left hand side of the graph].

The teacher extended her explanation by adding,

> So what you did was to put the majority of the points at one number to 10 there so that's where we are seeing these typical numbers where they are all grouped together, but then we also see that there's a group over here, but it's a smaller group.

The class continued to discuss the definition of median and how it divides the data set into two halves. We see evidence of student identity, agency and authority in this example, where the student was willing to share her graph with the whole class after posting it to a Padlet. She communicated her reasoning about how she came up with her solution and how it provided an answer to the task.

CONCLUSION

The analysis of the design and enactment of a lesson in a high school mathematics classroom showed how doing statistics aided by technology might support the development of positive mathematical identity, a sense of authority, and the growth of student agency. Student identity, authority, and agency support students to "use statistical evidence from analyses to answer the statistical investigative questions and communicate results with comprehensive answers," (NCTM & ASA, 2020, p. 19) as the GAISE II report recommends. Furthermore, designing lessons that consider student identity, authority, and agency is especially vital for under-represented populations, since minoritized students are given fewer opportunities to engage in ambitious mathematics tasks in comparison with Whiter and wealthier students (Billings, 1997). We learned that an effective and deliberate use of technologies can contribute to an environment that supports statistical thinking for all students. GeoGebra, an action technology, expanded access to, and engagement with, a task by allowing students to act on statistical objects (data displays and data points) and create meaning for key statistical concepts, in this case, mean and median. Padlet, a conveyance technology, allowed students to make their mathematical thinking public, post graphs that they created, and thereby build a collective understanding through discussion of shared objects.

We think it is unlikely for students to attain these understandings without the technologies employed. Action technology allowed students to experiment with many different values efficiently, something that would have been cumbersome and distracting otherwise. The visual displays of the data created almost instantaneously allowed students to make sense of the relationships between the values they entered, and then view changes effected by new inputs. The time-bound nature of the technology permitted experimentation with numbers that promotes understanding; this nascent understanding would be hampered by slower, more error-prone, and burdensome ways of calculation and display. The fact that students input their own data lent a sense of ownership over the ideas. The same can be said for conveyance technologies: sharing and comparing multiple data displays supported the growth of mathematical understanding, and it did so by putting student thinking front and center, literally and figuratively.

REFERENCES

Ben-Zvi, D., & Garfield, J. (2004). Statistical literacy, reasoning, and thinking: Goals, definitions, and challenges. In D. Ben-Zvi, & J. Garfield, (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 3-15). Kluwer Academic Publishers. https://doi.org/10.1007/1-4020-2278-6_1

Berry, R. Q. (2019, May). *Examining Equitable Teaching Using the Mathematics Teaching Framework*. *President's Messages,* National Council of Teachers of Mathematics. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Robert-Q_-Berry-III/Examining-Equitable-Teaching-Using-the-Mathematics-Teaching-Framework/

Billings, G. L. (1997). It doesn't add up: African American students' mathematics achievement. *Journal for Research in Mathematics Education*, *28*(6), 697-708. https://doi.org/10.5951/jresematheduc.28.6.0697

Chance, B., Ben-Zvi, D., Garfield, J., & Medina, E. (2007). The role of technology in improving student learning of statistics. *Technology Innovations in Statistics Education*, *1*(1), 1-26. https://doi.org/10.5070/t511000026

Daiga, M., & Driskell, S. (2021). Visualizing the arithmetic mean. *Mathematics Teacher: Learning and Teaching PK-12*, *114*(8), 607-615. https://doi.org/10.5951/mtlt.2020.0192

Dick, T. P., & Hollebrands, K. F. (2011). *Focus in high school mathematics: Technology to support reasoning and sense making*. National Council of Teachers of Mathematics.

Dykema, K. (2024). *The importance of data science.* National Council of Teachers of Mathematics. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Kevin-Dykema/The-Importance-of-Data-Science/

Edwards, M. T. (2001). *The electronic "other": A study of calculator-based symbolic manipulation utilities with secondary school mathematics students*. (Publication No. 3011050) [Doctoral dissertation, The Ohio State University]. ProQuest Dissertations Publishing.

Konold, C., & Higgins, T. L. (2003). Reasoning about data. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 193-215). National Council of Teachers of Mathematics.

Lane-Getaz, S. J. (2006). What is statistical thinking, and how is it developed? In C. Franklin, J. B. Garfield, & G. F. Burrill (Eds.), *Thinking and reasoning about data and chance: Sixty-eighth NCTM Yearbook* (pp. 273-289). National Council of Teachers of Mathematics.

Mokros, J., & Russell, S. J. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education*, *26*(1), 20-39. https://doi.org/10.5951/jresematheduc.26.1.0020

National Council of Teachers of Mathematics (NCTM) and American Statistical Association (ASA). 2020. *Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II: A Framework for Statistics and Data Science Education (GAISE II).* NCTM; ASA.

Peters, S. A., Bennett, V. M., Young, M., & Watkins, J. D. (2016). A fair and balanced approach to the mean. *Mathematics Teaching in the Middle School*, *21*(6), 364-375. https://doi.org/10.5951/mathteacmiddscho.21.6.0364

Sedlmeier, P. (2014). Technology for developing statistical thinking: A psychological perspective. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics* (ICOTS9). International Statistical Institute. https://icots.info/icots/9/proceedings/pdfs/ICOTS9_9F2_SEDLMEIER.pdf

Shaughnessy, M. (2007). Research on statistics learning and reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 957–1010). Information Age Publishing.

Shaughnessy, M. R., Chance, B. L., & Kranendonk, H.A. (2009). *Focus in high school mathematics: Reasoning and sense making in statistics and probability*. National Council of Teachers of Mathematics.

Suh, J. (2016). *Math tech-knowledgy for equity: Equity-centered Transformative Technology (EQTTech).* Onmason. http://mathtechknowledgy.onmason.com/intro/

Suh, J., Roscioli, K., Leong, K. & Tate, H. (2022). Transformative technology for equity-centered instruction. In E. Langran (Ed.), *Proceedings of Society for Information Technology & Teacher Education international conference* (pp. 1559-1567). Association for the Advancement of Computing in Education. https://www.learntechlib.org/primary/p/220920/