BRAZILIAN RESEARCH IN STATISTICS, PROBABILITY, AND COMBINATORICS EDUCATION: A LOOK AT THESES

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ABSTRACT

In this article, we first analyze the situation of statistics education in Brazil, within an international context of interest for this discipline. Secondly, we aim to answer the following research question: How has doctoral research in statistics, probability, and combinatorics education evolved in Brazil? For this purpose, we analyzed the Brazilian doctoral research production from 1994 to 2021. We identified 102 completed dissertations in doctoral programs over the last 28 years. Within this research context, we analyzed the graduate programs, advisors, research themes, grade levels and participants studied, theoretical frameworks, and the data analysis methodologies employed. Even though doctoral research has developed slowly and irregularly, there has been an increase in the number of studies, driven by the growth of graduate degrees offered in the last few years.

Keywords*: Statistics education research; Statistical, probabilistic, and combinatorial education; Brazilian dissertations; Teaching of statistics; Teaching of probability and combinatorics*

1. INTRODUCTION

In recent years in Brazil, several researchers have focused on statistics education. The fact that quantitative information is omnipresent points to the need to develop basic statistical skills and a culture of teaching statistics in society (Gal, 2002; Watson, 2006), including in formal learning institutions, and school curricula (Batanero, 2001). One goal of statistical education is the interpretation and understanding of statistical knowledge by students in compulsory and higher education and by citizens. This premise was highlighted in the International Conference on Teaching Statistics (ICOTS) held in 2018, in Kyoto, Japan, and in Rosario, Argentina, in 2022.

Statistics education is a relatively new area of investigation and scientific inquiry. With the recognition of its relevance to society today, there has been a focus on the pedagogical aspects of statistics education (Batanero, 2019). In recent years, there has been a growing worldwide interest in statistics education, reflected by the increase in the number of theses, articles, books, events, and research groups on the subject. There are researchers across the globe, including in Spain, Portugal, the United Kingdom, Israel, the United States, New Zealand, Australia, and Brazil. The latter is well represented in the international community, ranked second only to the United States in the number of dissertations produced.

Brazilian research has also been growing through international partnerships. We highlight the Hispano Brasileño de Educación Estadística Seminar, held remotely in 2020, through the collaboration of the Working Group on Statistical Education (WG12) from Brazil and the PAI–FQM 126 group, from the University of Granada, Spain. Another example was the INCREASE forums (International Forum for Early Career Researchers in Statistics Education) for discussion on statistics education, organized by Iddo Gal (University of Haifa, Israel), Mauren Porciuncula Federal University of Rio Grande [FURG], Brazil), and Carlos Monteiro (Federal University of Pernambuco [UFPE], Brazil) from October 2021 to May 2022. The monthly meetings were intended for researchers who recent doctoral graduates and had their doctoral dissertations approved in Brazil.

In Brazil, the WG12 (Working Group 12), created in 2000, aims to study and understand the teaching and learning of statistics, involving cognitive and affective aspects and the epistemology of concepts inherent to statistics. According to Lopes et al. (2010), the WG12 emerged at the International Seminar on Research in Mathematics Education (ISRME) and is linked to the Brazilian Society of Mathematics Education known as Sociedade Brasileira de Educação Matemática (SBEM, www.sbembrasil.org.br). Currently coordinated by Professor Ferreira Monteiro of the Universidade Federal de Pernambuco (UFPE), the WG12 has about 40 members belonging to research groups from different educational institutions. It has promoted several research activities in recent years, including the WG12 Forum in São Paulo, in 2018.

Within this context, this study aims to answer the following question:

> *How has doctoral research in statistics, probability, and combinatorics education evolved in Brazil?*

Given the growing number of investigations in statistics education in the country, we surveyed all Brazilian doctoral dissertations, analyzing the grade levels studied, themes and types of participants investigated, theoretical frameworks employed, theories used in the analysis of the data, and the main advisors of these studies. To contextualize the analysis, we offer a brief history of statistics education at a global and national level, quantify the undergraduate and postgraduate courses at a national level, and analyze the statistics content of the Brazilian curricular standards known as the *Base Nacional Comum Curricular* ([BNCC], Brazil Ministério da Educação, 2018) for all levels of compulsory education. We present the reflections arising from our analysis in the following sections.

2. THEORETICAL FRAMEWORK

In this study, a systematic review was used as the basis for the research survey. According to Newman and Gough (2020), there are different methods of systematic review by which educational researchers can expand their knowledge and understanding of the educational scenario. Dowd and Johnson (2020), in turn, emphasized that "the reason to publish systematic reviews of educational research is to communicate with people who may or may not be familiar with the topic of study" (p. 84). Therefore, understanding the state of knowledge on a given topic in a specific period is fundamental for the evolution of science, given that.

> … a systematic process of survey and description of the information about the research produced on a specific field of study, covering a certain location (place) and length of time. This information relates to the physical aspects of this production (describing where, when, and how many studies were produced over the period, and who the authors and participants of this production were), as well as their theoretical-methodological and thematic aspects. (Fiorentini et al., 2016, p. 18)

The literature review to determine the state of knowledge in a specific area is characterized by identification, registration, and categorization to promote reflection and synthesis on the scientific production of this area (Morosini & Fernandes, 2014). This study encompasses dissertations in statistical education, which, in the conception of Romanowski and Ens (2006), contribute to the construction of the state of knowledge in statistics education since a sector of the publications in this area was contemplated. Studies of this nature suggest a provisional conclusion, that is., they contribute to explaining the themes addressed in each area as a "balance of the knowledge produced" (Moraes & Peixoto, 2017, p. 325). Thus, they allowed us to identify, characterize, and analyze the uninterrupted movement of science.

3. EMERGENCE OF STATISTICS, PROBABILITY, AND COMBINATORIAL EDUCATION

To better situate our work, we first briefly describe the emergence of statistics education, and in the next section, we describe the situation in Brazil.

The first World Congress in Statistics was organized by Lambert Adolphe Jacques Quetelet (1796–1874) in 1853, in Brussels. Then, in 1885, the International Statistical Institute (ISI) was created to celebrate the jubilee of the London Statistical Society. From then on, the ISI established fundamental objectives for the advancement of statistical education, such as supporting the international statistical community, fostering research on statistical practices and statistical education, training young statisticians, and raising public awareness of the role of statistical procedures in everyday life (ISI, 2017b).

Within its 130 years of history, the ISI has contributed to the demands of statistics, bringing together researchers from different areas, and promoting, as part of its mission, the understanding and development of sound statistical practices around the world (ISI, 2017a). This association became effectively and systematically involved with statistical education after the Second World War, when it considered the development of this area as promising, contributing to solving the economic and social problems of countries affected by war or in different stages of development.

In 1949, the United Nations Organization (UNO) approved a resolution that requested the United Nations Educational, Scientific, and Cultural Organization (UNESCO) and the ISI to respond appropriately to the advancement of statistics education on an international scale. As a result, in the same year, the Committee on Statistics Education was established (Philips, 2002). The Committee sponsored the first roundtable conference in 1968. Since then, there have been 13 roundtables, the first seven organized by the Committee and the last six, as of 1996, held by the International Association for Statistical Education (IASE, https://iase-web.org/Conference_Proceedings.php).

In 1967, a joint committee was formed between the American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM) to establish a curriculum for K–12 (Garfield et al., 2008), which corresponds to the compulsory years of education in Brazil. These advances in statistical education are due to the collaboration between statisticians and mathematics educators. One example of such advancement is the Quantitative Literacy Project (QLP), a decade-long activity between the NCTM and the ASA that focused on materials development for the teaching of data analysis and probability.

In 1991, the ISI created a section to develop statistics education, the IASE, which took over the activities of the education committee. Among the associations belonging to the ISI, the IASE is the one responsible for the promotion, support, and improvement of statistical education at all levels. To this end, it seeks, through international cooperation, to foster and communicate ideas and research results that can contribute to teaching and learning. In this regard, Batanero (2019) pointed out that the purpose of the IASE is to promote discussion among those involved in statistical education, advance statistics as an independent area of study and research, and lead and promote statistical education, particularly in developing countries.

The first journal devoted to the subject, the *Journal of Statistics Education* (JSE), appeared in 1993. In 2002, the *Statistics Education Research Journal* (SERJ) was launched. While the former focused on the teaching of statistics, the latter focused exclusively on the publication of research in statistics education (Garfield et al., 2008). Even today, the number of journals devoted to statistics education is small, compared to the hundreds of mathematics education journals.

In Brazil, the first milestone in statistics education was the publication of education guidelines that were intended to communicate the principles of curriculum reform, and guide the work of teachers, through the *National Curriculum Parameters* (NCP), known as *Parâmetros Curriculares Nacionais* (PCN), in 1998. Despite the availability of the curriculum framework, there was little advancement in teaching pedagogies as the guidelines did not convey an effective teaching practice. In addition, there was no alignment between the *National Curriculum Parameters* and the curricula of undergraduate mathematics teaching degrees used to train prospective schoolteachers to teach statistics and probability in the compulsory education years, as the degree programs did not include statistics and probability in their curricula.

The teaching of statistics has advanced little since the introduction of the NCP. According to Viali (2008), in a survey of 125 undergraduate curricula throughout Brazil, less than 3% of the courses

included the teaching of combinatorics, statistics, and probability. In most curricula, there was only one four-credit course (60 hours), usually shared with other undergraduate programs. This finding reveals the need to promote better training so that teachers can develop skills related to the learning and teaching of statistics in primary and secondary education.

4. STATISTICS EDUCATION IN BRAZIL

The teaching of statistics in Brazil is included at all educational levels, from compulsory to higher education. Despite numerous previous demands, the teaching of statistics and probability in Brazil only became part of the law that regulates education (Education Guidelines and Basis Law) in 1996, with the creation of a series of legal documents that made its teaching mandatory in school curricula throughout the national territory. Currently, based on these guidelines, norms, and resolutions, Brazil has collaboratively consolidated, through the BNCC (Brazil Ministério da Educação, 2018), the minimum requirements of statistics and probability in primary and secondary education.

The BNCC (Brazil Ministério da Educação, 2018) establishes the knowledge, competencies, and skills that all students should develop during their compulsory education years (from 4 to 18 years of age). In secondary education, its homologation occurred by the end of 2018, and its implementation took place in 2022. Brazilian compulsory education is structured sequentially, as shown in Figure 1.

<table>
<tr><td colspan="12">Compulsory Brazilian education</td></tr>
<tr><td colspan="2" rowspan="2">Child education</td><td colspan="7">Elementary school</td><td colspan="3" rowspan="2">High school</td></tr>
<tr><td colspan="4">Early years</td><td colspan="3">Final years</td></tr>
<tr><td>Level 4 (age – 4/5)</td><td>Level 5 (age - 5/6)</td><td>1st grade (age – 6/7)</td><td>2nd (age – 7/8)</td><td>3rd grade (age - 8/9)</td><td>4th grade (age – 9/10)</td><td>5th grade (age – 10/ 11)</td><td>6th grader age – (11/ 12)</td><td>7th grader (age – 12/ 13)</td><td>8th grade (age – 13/ 14)</td><td>9th grade (age – 14/ 15)</td><td>1st year (age – 15 /16)</td><td>2nd year (age – 16/ 17)</td><td>3rd year (age – 17/ 18)</td></tr>
</table>

Figure 1. Stages of compulsory Brazilian elementary education.

In elementary school, Probability and Statistics is one of the five thematic units, along with Numbers, Algebra, Geometry, and Quantities and Measures. All the themes are linked to the pedagogical knowledge of mathematics to promote mathematical literacy. Thus, the BNCC in elementary school divides mathematics into five thematic units, Probability and Statistics being one of them and emphasizes that mathematics is not only restricted to the quantification of deterministic phenomena but also studies the uncertainty arising from random phenomena. In contrast, combinatorics is included in Numbers in elementary school. For secondary school, the final stage of compulsory education, the BNCC establishes the articulation between the thematic axes proposed for elementary school, grouping fundamental ideas, namely: "variation and constancy; certainty and uncertainty; movement and position; relationships and interrelationships" (Brazil, 2018, p. 520).

Secondary school is the final stage of compulsory education, during which time the curriculum focus is on the “consolidation and strengthening of the knowledge acquired” (Brazil Ministério da Educação, 2018, p. 464). The secondary curriculum developed from the BNCC includes formative itineraries, that is, courses that students can choose to deepen their understanding of specific areas of study. These courses are organized taking into consideration the regional and local curricular subject areas and their arrangements.

Mathematics is one of the four main areas of the national curriculum: mathematics, and its technologies, along with languages and its technologies, nature sciences and its technologies, and human sciences and its technologies (Brazil, 2018). In this regard, the teaching and learning of probability and statistics are integrated with other learning possibilities proposed along with the development of these five specific competencies and associated abilities. The curriculum suggests an interdisciplinary approach to technology and problem-solving. This document highlights that counting problems, elements of statistics, and concepts of probability are intrinsic to various forms of social communication. Although work with statistics, probability, and combinatorics begins in Kindergarten,

it is in the early years that the BNCC explicitly describes the skills and competencies to be developed on these topics.

Regarding statistics, the BNCC details that during the early elementary school years, students should have the opportunity to work with news reports as well as plan and execute a sample survey. It also emphasizes the importance of communicating the results of surveys and constructing graphs. For the final primary years, the students are expected to get involved in scientific initiation activities and construct descriptive statistical reports. In secondary school, the emphasis is on the interpretation, analysis, and construction of different types of graphs and tables, including data obtained from sample surveys, with or without the use of software. At this stage, problems involving calculations and interpretation of measures of central tendency and spread are proposed (Brazil Ministério da Educação, 2018).

As for probability, the BNCC's main objective in the early years is to develop the students' understanding that not all phenomena are deterministic. To this end, the notion of randomness is addressed through situations that can be classified as certain, impossible, or probable. In the final years, this document stresses the importance of developing random experiments and simulations. In secondary school, probability permeates different areas of mathematics, and the learning approach highlights the idea of uncertainty to interpret predictions, identify different sample spaces, and calculate the probability of random events. According to the BNCC, the progression of knowledge is made by improving the ability to enumerate the elements of the sample space, which is also associated with counting problems (Ministério da Educação, Brazil, 2018). The counting problems for the early primary years (7 to 11 years old) are at first restricted to cases in which it is possible to enumerate all possibilities, employing schemes or diagrams. For the final primary years (11 to 15 years old), the multiplicative principle is applied in the construction of the sample space when working with probability. In secondary education, this topic is foreseen in the development of skills related to groupings of elements that can be ordered or not by applying multiplicative and additive principles (Ministério da Educação, Brazil, 2018).

The teaching of statistics in primary education is often done by teachers who have not received adequate training, since most mathematics undergraduate programs only offer one statistics course (4 credits), which involves notions of probability. Few teacher education curricula include a specific subject involving the pedagogy of statistics or statistics education (Viali, 2008). Thus, Brazil has weaknesses concerning the teaching of statistics and probability, arising from incipient teacher education. Santos (2017, p. 215) corroborated this idea by stating that "there is still, in Brazil, a precariousness observed in the training of teachers in statistics education."

More recently, Buehring and Grando (2019), in an article involving doctoral and master's research, focused on the early years of Brazilian elementary education, asserted that teachers with training in pedagogy have neither studied statistics in their initial undergraduate education nor their continued development. In this regard, Coutinho et al. (2019) emphasized the need for teachers and students to be statistically literate so that they do not work on "content in an automatic and uncritical way" (p. 2).

5. METHODOLOGY

This study is a systematic literature review of the Brazilian dissertations produced in statistics education until the end of 2021. Twenty-eight years have passed since the first doctoral dissertation was produced in Brazil. During this time frame, 102 dissertations were identified. The list of dissertations accessed is listed in the Appendix.

The review of the doctoral dissertation production was carried out essentially via the Internet, accessing theses stored digitally. Although not all the theses were fully digitized, finding their abstracts or references was possible. The consultation sources included the CAPES dissertation bank, the Brazilian Digital Library of Theses and Dissertations, Biblioteca Digital Brasileira de Teses e Dissertações (BDTD), and the ISI repository. To search, the keywords "Statistical Education", "Statistics Teaching", "Probability Teaching", "Combinatorics Teaching", and "Stochastics Teaching" were used. The year, place, title, author, graduate program, and supervisor were also identified. We also verified the participants involved, the themes and objects of study of the papers, and the topics categorized as statistics, probability, combinatorics, stochastics, or some combination such as statistics and probability or combinatorics and probability.

6. EXPLORING DATA ON DISSERTATIONS

From the first thesis in 1994, 102 works were found until the year 2021; therefore, the period involves 28 years, which corresponds to an average of approximately 3.6 theses per year. Except for the years 1995, 1997, and 1998, when there were no defenses, all the other years presented at least one d in the area, as shown in Figure 2.

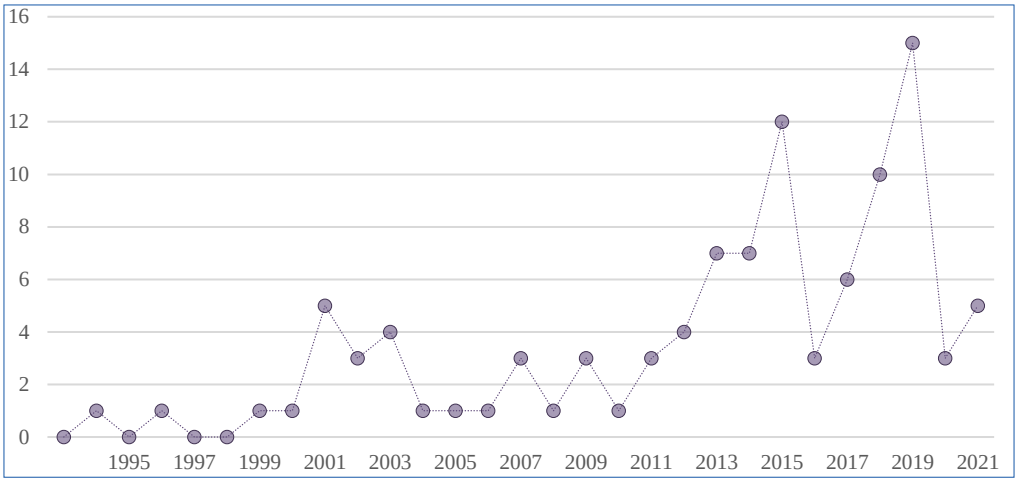

Figure 2. Brazilian production of theses in statistics education in the period 1994–2021.

The beginning of the new millennium showed an increase, with the production of five dissertations in 2001. This growth, however, was not sustained, a fact that can be observed in 2004, 2005, and 2006, in which production remained constant at only one dissertation per year. Considering the growth in the number of doctoral programs in mathematics education or science and mathematics during this time, we expected an increase in dissertations, particularly due to the publication of the NCP in 1997.

6.1. THE PROGRAMS

There is currently no specific doctoral program in statistics education in Brazil. About 58.8% of the dissertations written in this area of study come from education or mathematics education programs. Figure 3 illustrates the distribution of dissertations considering the type of postgraduate program (PGP).

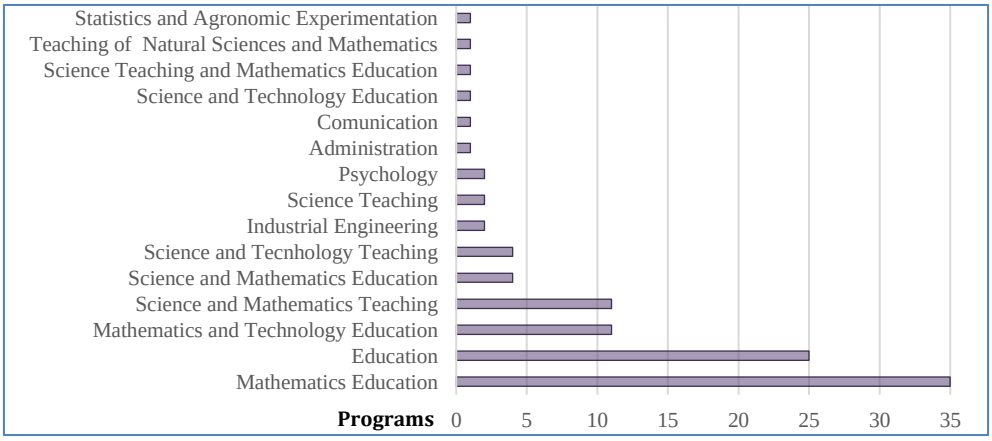

Figure 3. Titles of the programs that present dissertations in statistics education.

Education programs are the oldest in the country and represent a good portion of the dissertations, with approximately 24.5% of them coming from these programs. These programs, however, have been losing space to newer studies such as mathematics education, science and mathematics education, and science and mathematics teaching, which represent about 60% of the dissertations. Although the programs have different names, they all include mathematics.

Some of the dissertations, in particular the older ones, were part of programs in areas different from education, since at the time there were a reduced number of programs in the country, making access difficult. For example, dissertations were written as part of production engineering programs, communication, business administration, and psychology programs, as well as one program exclusively in science and another in science and technology. It is worth mentioning that there was a dissertation focused on statistics education written in the PGP in statistics and agronomic experimentation.

6.2. SUPERVISION

In Brazil, all those who teach in higher education are called professors. In most educational institutions, both public and private, the teaching career is formed, in general, by five categories of professors: teaching assistants (beginning of the career), assistants, adjuncts, associates, and full professors (top of the career). Professors with a doctorate start their careers as adjuncts and can reach the full professor category with scientific production evaluated by peer review or by writing an original thesis. All professors who teach at the graduate level must be doctors (Ph.D. qualification) with an established scientific research record. In general, to be a graduate teacher, it is necessary to obtain accreditation, which must be renewed periodically.

The 102 Brazilian dissertations analyzed had 57 different advisors. In Brazil, most of these professionals supervised studies in mathematics education, an area with a higher research production when compared to statistics education. Only 15 of all the advisors supervised more than one study in statistics education, while the other 42 supervised only one dissertation. That is, 73.7% of the supervisors had a single supervision in the area. This shows that there is not a body of professors focused exclusively on research in statistics teaching. It is noteworthy that, with rare exceptions, these supervisors are mathematics graduates. Some of them are lecturers in statistics and probability, but none have a statistics degree.

Three researchers stand out for being responsible for 25 dissertations, corresponding to 24.5% of Brazilian studies in statistics education at the doctoral level. The three researchers with the highest production were Cileda de Queiroz e Silva Coutinho with 10 supervisions, followed by Celi Aparecida Espasandin Lopes with eight supervisions, and Maria Lucia Lorenzetti Wodewotzki with seven supervisions.

6.3. RESEARCH TOPICS

We identified the topics of the theses from the title, abstract, and, when necessary, from reading the text. For this study, we considered the productions in probability and combinatorics in the context of statistical education. Although probability is not exactly statistics, as is the case with combinatorics, they are related and are indispensable subjects for the teaching of stochastics.

Out of the 102 dissertations, 64 (62.7%) had statistics as the main theme, and the most explored topics were descriptive statistics with measures of central tendency and the mean. Probability was the subject of 18 (17.6%) dissertations, and the basic concepts of probability, sample space, and events were the most investigated topics. Combinatorics was the theme of eight (7.8%) dissertations, two of which investigated both combinatorics and probability. Statistics and probability were the subjects of seven (6.9%) dissertations, in which the basic concepts of both statistics and probability were investigated without, however, relating them to each other specifically. The relationship between statistics and probability, that is, stochastics, was the focus of four studies (3.9%). This information is shown in the graph presented in Figure 4.

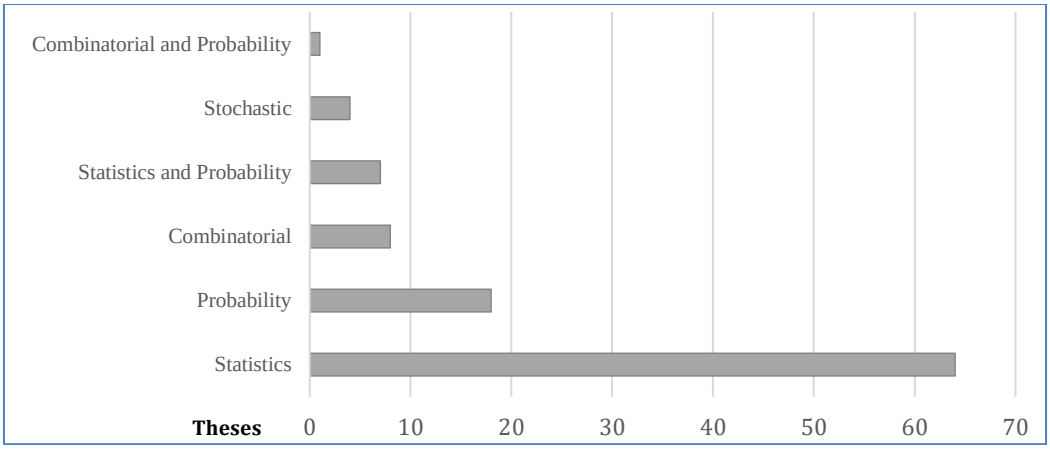

Figure 4. Subjects covered in statistical education dissertations.

Although most of the research addressed higher education, no theses were found involving the teaching or learning of more complex subjects, such as sampling, estimation, hypothesis testing, correlation, regression, analysis of variance, and multivariate statistics (see Figure 4). The only counterpoint was a study on quality control. The most covered subject was descriptive statistics, with the main topics being the analysis and interpretation of graphs and tables, and measures of central tendency and variability. It is important to mention that no research involving descriptive measures, such as asymmetry and kurtosis, was found.

The themes were varied. As for what was investigated, we found that a good number of the dissertations (24, 23.5%) dealt with the training and professional knowledge of teachers at all levels. This topic was followed by 22 studies on learning statistics (including all levels), or 21.6% of the research projects. In third place was teaching statistics, with 14 dissertations (13.7%). Note that we make a distinction between the teaching and learning of statistics, combinatorics, and probability. Other popular themes included the learning of probability (9 dissertations; 8.8%) and graphical representations (7 dissertations; 6.9%). This last topic could have been included in the teaching/learning of statistics; however, we highlight it because it was the subject of several studies. Finally, four research studies centered on textbooks for compulsory education (3.9%).

Our analysis is comparable to that of Silva et al. (2017). These authors analyzed the publications related to statistics education in the *Bulletin of Mathematics Education*, known as Boletim de Educação Matemática (*BOLEMA)*, from 2006 to 2015, one of the most renowned journals of mathematics education in Brazil. The authors found that most of the research they analyzed in this area dealt with teaching, teaching resources, and educational frameworks. A further recurrent theme concerns the training and professional knowledge of teachers at various levels of education, corresponding to 23.5% of all dissertations. The investigation of Silva et al. (2017) verified that the articles addressed teacher training and aimed to overcome teachers' difficulties regarding initial education and further development. Other Brazilian researchers, such as Porciúncula et al. (2018), Schreiber and Porciúncula (2019), and Tinti and Lopes (2021), have investigated this theme. In addition, teacher training was also highlighted at the Seminario Hispano Brasileño de Educación Estadística in 2000. As a result of this meeting, the text, *Investigações Hispano-Brasileiras en Educación Estadística* [Hispano-Brazilian investigations in statistical education], was published (Ribeiro & Paván (2000).

6.4. EDUCATION LEVELS AND RESEARCH PARTICIPANTS

Most of the research (47.1%) was carried out within higher education, with a focus on teachers, students, statistics courses, or explaining the dropout rates in statistics courses. Elementary education, the second level of education with the largest number of dissertations in the area, concentrates its studies on the conceptions and training of practicing teachers, in addition to issues addressed in the statistics, probability, and combinatorics curricula, as shown in Table 1.

Table 1. The level of education investigated

Level	Dissertations	%
Undergraduate	48	47.1
Basic Education (Primary school)	38	37.3
Child Education (Kindergarten)	11	10.8
Youth and adult education	3	2.9
High School	1	1
Graduate students	1	1
Total	102	100

Regarding the study participants, most of them were students and teachers. In higher education, students from engineering courses had the highest concentration of participants. Other studies involved students from business administration, social sciences, economics, statistics, and mathematics. Regarding compulsory education, it was observed that the major focus was on teachers. This information can be seen in greater detail in Table 2.

Table 2. Research subjects in the statistical education theses

Research Subjects	Dissertations	%
Undergraduate students	23	22.5
In-service primary school teachers	12	11.8
Primary school students	11	10.8
Prospective mathematics teachers	8	7.8
In-service child education (Kindergarten) teachers	7	6.9
In-service university professors.	7	6.9
High school students	6	5.9
Child education students	5	4.9
In-service mathematics teachers	5	4.9
In-service high school teachers	5	4.9
Textbooks	4	3.9
Youth and adult education students	3	2.9
Other participants	6	5.9
Total	102	100.0

Dissertations involving “other participants” included the history of the Brazilian Institute of Geography and Statistics, known as Instituto Brasileiro de Geografia e Estatística (IBGE), the teaching of statistics in a specific São Paulo institution, the biography of the statistics Professor Frederico Pimentel Gomes, evaluation of a statistics course in post-graduate and adult education known as Educação de Jovens e Adultos (EJA), and a documentary survey of the history of statistics.

6.5. THE THEORETICAL FRAMEWORK AND DATA ANALYSIS METHODOLOGY

We examined all the studies to identify the theoretical framework as well as the data analysis methodology. In the dissertations, 42 different theoretical approaches were identified, but in eight of them (7.8%), we were unable to identify the approaches. The results are presented in Table 3.

Table 3. Theoretical framework

Theoretical Reference	Dissertations	%
Statistical and Probabilistic Literacy by Gal	18	17.6
Anthropological Theory of Didactics	9	8.8
Shulman's Pedagogical Content Knowledge	7	6.9
Ausubel's Meaningful Learning	6	5.9
Statistical Literacy, Statistical Reasoning, and Statistical Thinking	6	5.9
Vergnaud's Conceptual Fields Theory	6	5.9
Critical Statistical Education	4	3.9
Cognitive Psychology	3	2.9
Onto Semiotic Approach Theory of Mathematical Knowledge and Instruction	2	2.0
Unidentified	8	7.8
Others	33	32.4
Total	102	100

It should be noted that these theories are not normally used in isolation. Many studies used more than one theoretical framework. In Table 3, we highlight the most relevant theory in each study. Other theories or approaches—33 in all—appeared in only one dissertation. Among them, Tardiff's Teaching Knowledge (Berlikowski, 2018), Kuhn's Paradigmatic Perspective (Wada, 1996), Wenger's Social Theory of Learning (Pamplona, 2009), Robert's Expected Knowledge (Bifi, 2009), Bryant and Nunes' Cognitive Demands (Brito, 2019), Fischbein's Probabilistic Intuition (Oliveira, 2013), Freire's Problematizing Education (Lopes, 2003), and Vygotsky's Cultural-historical Approach (Pamplona, 2009) stand out.

About 40% of the dissertations did not specify the method used in the data analysis, as shown in Table 4. These were older dissertations. Further, this issue was observed both in the identification of the theoretical framework and in the methodology of data analysis.

Table 4. Analysis methods

Theoretical Reference	Dissertations	%
Did not use a specific method	42	41.2
Descriptive analysis	20	19.6
Textual discourse analysis	7	6.9
Content analysis	6	5.9
Triangulation	5	4.9
Narrative	5	4.9
Onto semiotic Approach	3	2.9
Discourse Analysis	2	2.0
Statistical Implicative Analysis (SIA)	2	2.0
Anthropological Theory of Didactics	2	2.0
Others	8	7.8
Total	102	100.0

Statistical (descriptive) analysis, followed by narrative and triangulation, was the main method used in the theses analyzed. Other theses had unique methodologies for data analysis: Chevallard's Praxeological Analysis (Melo, 2018), Block's Regressive Method (Poubel, 2013), and Josep Gascón's Didactic Models (Pinheiro, 2015), are some examples.

7. NATIONAL AND INTERNATIONAL BRAZILIAN RESEARCH PRODUCTION

There is a high asymmetry in the research theses in statistics education in the national territory. Most of the dissertations were produced in the state of São Paulo, 64 (52.7%). The state of Pernambuco comes second, with 16 dissertations (15.7%). In third place, we found Paraná and Rio Grande do Sul, both with six dissertations (5.9%) each. Santa Catarina had four dissertations (3.9%), followed by Minas

Gerais with three, Espírito Santo with two, and Rio Grande do Norte with only one publication. As shown in the following figure (Figure 5).

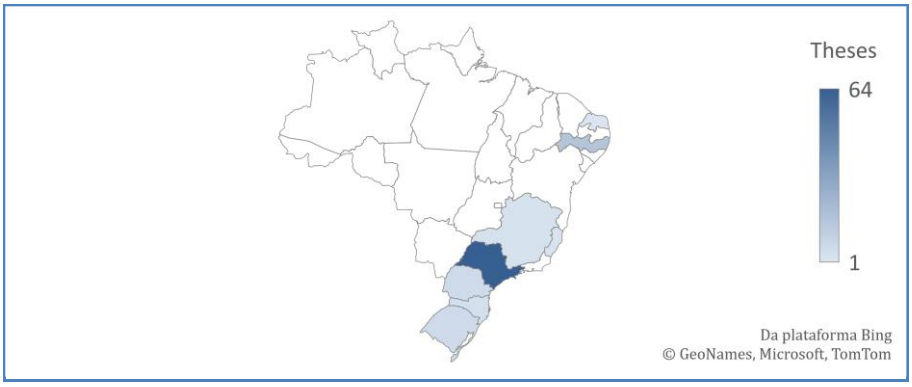

Figure 5: Geographical distribution of theses in statistics education in Brazil.

This inequality in research production is justified by the generous size of the country, as well as the social and economic inequalities among the different regions. This can be seen in Figure 5, which shows the number of productions in each of the 26 Brazilian states. The states colored in gray were not involved in research in the subject area of the current analysis. Interestingly, this same imbalance can be observed when we extend the scope to other countries, as shown in Figure 6.

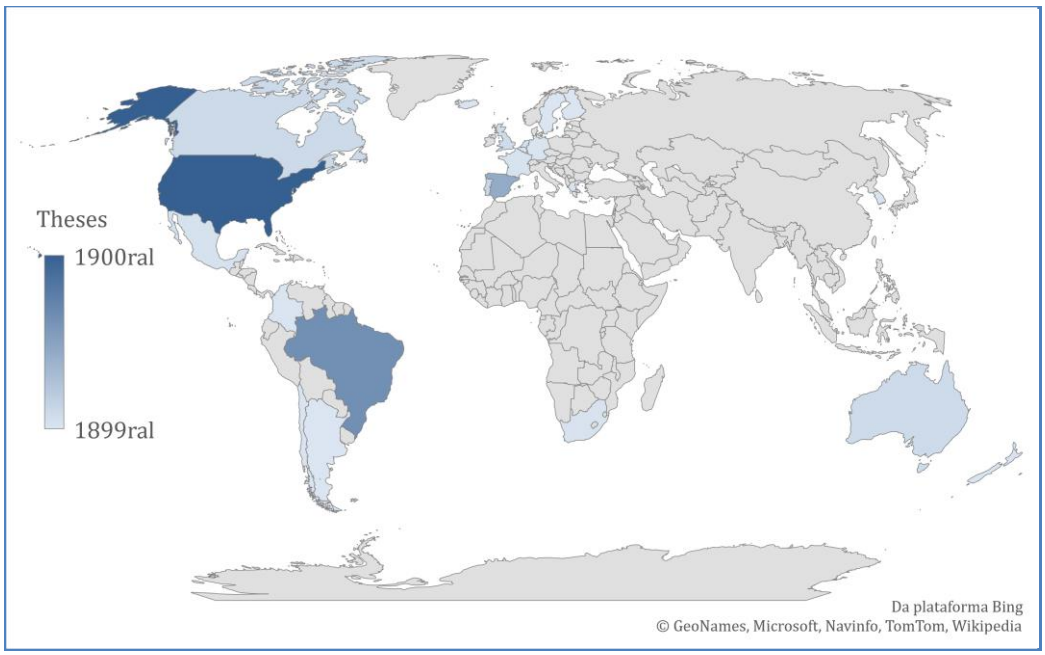

Figure 6: Geographical distribution of theses in statistics education worldwide.

When we turn to international production, we observed that the United States has the largest world production, with 158 dissertations (36.1%), followed by Brazil, with 102 (23.3%), Spain with 69 (15.8%), Portugal with 20 (4.6%), Canada with 15 (3.4%), Australia with 14 (3.2%), United Kingdom with 13 (3.0%), Netherlands with eight (1.8%), Mexico with six (1.4%), France with five (1.1%), South Africa with four (0.9%), and South Korea and New Zealand with three (0.7%) each. We also have the following countries with only two dissertations (0.5%) identified: Germany, Chile, Colombia, Greece, Iceland, and Israel. The following countries presented only one dissertation: Argentina, Belgium, Finland, Puerto Rico, Singapore, and Sweden, totaling 438 works in the area. The dissertations and the countries identified here were obtained through a thorough and exhaustive search on the worldwide

web, using the keywords Statistics Education, Statistics Teaching, Probability Teaching, Combinatorics Teaching, among others. The databases used were the CAPES Theses and Dissertations Catalog and the Brazilian Digital Library of Theses and Dissertations in Brazil, SciELO in Latin America, ProQuest Dissertations & Theses in the United States, Open Access Theses and Dissertations, DART-Europe E-theses Portal, Europe, ETHOS e-theses Online Service, United Kingdom, EBSCO Open Dissertations, Recent Doctoral Dissertations in statistics education at IASE, among others. In addition, publications of research groups in the area were consulted, as well as websites of professors who publish on the subject, such as the Grupo de Investigación sobre Educación Estadística of the University of Granada, Doctoral Study in Statistics Education at the University of Auckland, GrEF- Study Group on Statistical Education in Primary Education, IMEF- Center for Statistical Education, IEEP- Research Group on Statistical and Probabilistic Education, among others.

8. DISCUSSION

The analysis of Brazilian doctoral dissertations in the area highlights that about 96% of the dissertations were produced after the Brazilian government published the NCP, in 1997. Moreover, this publication, due to international trends, boosted Brazilian research in the area. This reality is positive, as it shows an attempt to expand investigations focused on the teaching and learning of statistics, probability, and combinatorics. We observed, however, that such studies did not impact the classrooms of Brazilian schools directly, potentially because of the lack of statistical training of teachers. Lopes e Souza (2016) corroborated this view by stating that curricular changes come from scientific research, but these do not guarantee transformations in educational practice. In everyday school life, curricular guidelines are not always implemented, due to different contexts—social, economic, educational, administrative—in which the school is inserted and the difficulty in putting curricular and pedagogical changes into practice (März & Kelchtermans, 2013).

Most Brazilian doctorate research has concentrated its studies on higher education (undergraduates). This shows a need for more research at the compulsory primary and secondary education levels since the BNCC determines that the teaching of statistics, probability, and combinatorics should begin in the first year of primary school (6–7 years-old). Regarding combinatorics, we only found a few studies on the subject, which indicates the need to expand research in that area.

During the review process, we found diversity in the theoretical frameworks adopted in doctoral studies. Nonetheless, most of the theses analyzed based their theoretical framework on the statistical and probabilistic literacy of Gal (2002, 2005). According to Gal (2021), numerous studies reported in conference proceedings organized by IASE address statistical literacy. The author claimed that "discussing needs and directions for developing statistical literacy is of special importance in the Brazilian context" (p. 44).

In addition, there is plenty of use of mathematics education theories. The reason may be that the advisors, in the majority, have trained in these areas and end up being responsible for advising on theses on statistical education. This scenario is explained by the fact that statistical education is a new area of knowledge. Additionally, there is a need to expand the theoretical approaches of this field, beyond mathematics education.

Bearing in mind the Brazilian territorial size, the diversity of context favors the variety of works and the collaboration among researchers. However, it also increases the challenge of expanding investigations, as many are concentrated in some states in the southern, southeastern and northeastern regions. Consequently, this circumstance reveals the need for more financial investments in training researchers in other regions or providing incentives to research in those regions.

9. CONCLUSION

The analysis of Brazilian doctoral dissertations in statistical education aimed to answer the following question: How has doctoral research in statistics, probability, and combinatorics education evolved in Brazil? Considering the analysis, we concluded that doctoral research has progressed slowly and irregularly. It is worth mentioning that most of the Brazilian dissertations come from education programs or mathematics education programs since there is not yet a specific Ph.D. program in statistics education available.

Even if irregularly, the production of dissertations in statistics education is advancing in Brazil, due to the growth of doctoral programs in the areas of Science and Mathematics Teaching/Education or Mathematics Teaching/Education. Therefore, research production in these programs in these areas is expected to grow significantly in the coming years.

The growth in research in statistics education over the past three decades has resulted in the inclusion of statistics in the elementary and high school curricula of many countries. There are, however, many challenges to be faced along this journey (Vásquez, 2020). To address this situation successfully, it is necessary to develop research in this area of knowledge. Therefore, further research should be undertaken in the coming years to verify if the growth of Mathematics or Science Education/Teaching programs has promoted such an advance in statistics. The difficulty in finding dissertations in the databases was a limiting factor, given that some of them were not incorporated into digital systems. Furthermore, several dissertations do not present, in their abstract, the data sought here, demanding a more in-depth reading.

The relevance of this investigation is justified by the fact that "the task of communicating discoveries and recommendations must be carried out" as indicated by Dowd and Johnson (2020, p. 70), thus making it possible to compile, in a study, these theses and their respective data to promote the research and dissemination of information on statistical education in Brazil.

The communication network between school and University must be expanded, to promote the improvement of pedagogical practices in the field of statistical education, as advocated by Porciúncula et al. (2018). At the same time, universities need to understand the context and realities of schools so that their research meets the school's demands. Therefore, we suggest, for future research, studies focused on teaching practices in basic education to enhance the skills of both current and future teachers.

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APPENDIX

Statistical, probabilistic, and combinatorial education theses analyzed and defended, in Brazil, until Dec/2021. All these are in Portuguese and contain an English abstract. Most of them can be found in the CAPES catalog of theses and dissertations (https://catalogodeteses.capes.gov.br/catalogo-teses/#!/).

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Observation. In addition to the 102 dissertations defended in Brazil, there are four more theses by Brazilian teachers that were defended abroad. One in France and three in Portugal. These were not included in the analysis.

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