COGNITIVE AND SOCIOCULTURAL ASPECTS OF SECOND-GRADE STUDENTS' INFORMAL INFERENTIAL REASONING

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ABSTRACT

This paper reports on a study concerning the social nature of young students' informal inferential reasoning. Employing inferentialism as a background theory, we examine cognitive and sociocultural aspects of reasoning that arose during group discussions as well as trace relations between those aspects. Following a design experiment approach, we analyzed the discussion that emerged while a group of second-grade students was working on a carefully constructed inferential task. The results illustrate the dynamic interplay between cognitive and sociocultural aspects of informal inferential reasoning during students' attempts to reach a conclusion while working in a group.

Keywords: *Informal inferential reasoning; Inferentialism; Young learners*

1. INTRODUCTION

Due to the increased availability of data, statistics is often portrayed as a means of adding credibility to the information shared in, for example, advertising, journalism, or casual discussions (Ben-Zvi & Garfield, 2004). As a consequence, being able to properly assess evidence and claims based on information is an important skill not only for those dedicated to science but also for informed citizens in today's societies. Under these circumstances, scholars in statistics education have started to support the integration of important social and political issues into the school curriculum, with the aim of fostering students' meaningful participation in their local and global communities. The basic argument behind this tendency is that statistically literate students should not be simple consumers or producers of data-based arguments but should be able to consider how these arguments shape the reality in which they live (Weiland, 2017).

Statistical inference is a component of statistical literacy that enables students to form and evaluate decisions and conclusions in everyday life. Formal statistical inference entails probabilistic computations such as parameter estimation and hypothesis testing in order to create and evaluate probabilistic generalizations from data. Makar and Rubin (2018) examined the research findings related to these statistical procedures and concluded that many university students face difficulties in mastering formal statistical inference. At the primary school level, statistics education is mainly focused on skills such as constructing diagrams or calculating averages (Ben-Zvi & Amir, 2005). Due to this narrow and limited interpretation of statistics in elementary education, researchers and curriculum designers have advocated for a broad interpretation of statistics in school mathematics so that even young students gain access to statistical procedures and concepts such as inference through familiar experiences.

In order to characterize young students' inferences, the term informal statistical inference (ISI) has been employed to denote probabilistic generalizations from data (Makar & Rubin, 2009). The usage of the word "informal" indicates that ISI is conceptualized in a manner broader than formal statistical tests, and students are not expected to draw upon formal statistical procedures to infer conclusions from data (Makar et al., 2011). While investigating a sample of data, students may draw upon prior knowledge, personal experiences, or hypotheses in order to explain their reasoning (Gil & Ben-Zvi, 2011). Engaging students in inferential activities is hypothesized to be not only a way of preparing them for formal

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inference but also an opportunity to connect statistical procedures such as prediction and estimation to everyday activities and an attempt to unify statistics and probability (Makar, 2016). As Estrella (2018) pointed out, educating students as citizens so that they can make data-informed decisions should begin at an early age. This way, young students might become more competent in decision-making by developing skills such as critical thinking and creativity so that the skills could possibly become "habits of mind" by the time students become active members of society.

In recent decades, topics related to students' ISI and informal inferential reasoning (IIR) have been studied in educational research. Researchers have examined the nature of students' ISI (Makar & Rubin, 2009), the role of context in the development of students' IIR (Makar & Ben-Zvi, 2011), and the elements supporting this development (Leavy, 2017). Some of these studies focus on young students' inferential experiences. For example, Makar (2016) attempted to understand and support the emergence of ISI in nursery school and first-grade students. Through design research, she noticed that students tended to employ practices such as making predictions, using data as evidence, and articulating uncertainty. Makar also proposed a number of strategies that supported students' informal reasoning such as permitting young students to record and organize data in their personal ways. Adopting a similar research scope, Leavy (2017) suggested that elements such as placing data investigations within interesting contexts, listening to peers' reasoning, and questioning students adequately supported students' emerging understandings of statistical inference. These studies concluded that young students possess conceptual resources that, with adequate support from teachers and curricular design, can be extended toward more sophisticated forms of reasoning (Makar & Rubin, 2014).

Since the early 1980s, the mathematics education research community has turned its attention to viewing meaning, thinking, and reasoning as products of social activity and not as objects emerging from and within the minds of decontextualized individuals (Valero, 2004). Similarly, a substantial number of statistics education studies attend to social aspects of students' IIR. For example, Gil and Ben-Zvi (2011) focused on sixth-grade students' explanations as social constructs during IIR processes. Pfannkuch (2011) investigated the role of context in developing IIR, with both grade 10 students and their teachers perceived as learners. In alignment with this work, our study aims to investigate how students' IIR emerges socially in mathematics classrooms, that is, through students' linguistic interactions and mutual evaluations. To do so, we employ the semantic theory of inferentialism that, according to existing literature, fits well with the idea of statistical inference (Nilsson et al., 2018) and offers a lucid view on the relation between the individual and the social (Schacht & Hußmann 2015).

2. THEORETICAL BACKGROUND

2.1. ISI AND IIR

ISI is conceptualized as a claim that extends beyond the data, in which the data serve as evidence to support a conclusion expressed with uncertainty (Makar & Rubin, 2009). There are three ideas that can be seen as underpinning ISI (Makar & Rubin, 2009). The first key feature is generalization beyond the data, that is, a statement that extends beyond a given sample. Konold and Pollatsek (2002) argued that while examining data in statistics, one needs to search for the "signal" (some kind of pattern in a distribution) inside the noise (variability) of the data. This way, the validity of a statement against available data can be informally evaluated. The second critical ISI feature is the use of data as evidence. Everything that can support a statement in a given context can be considered evidence (Schindler & Seidouvy, 2019). As a result, evidence is both contextual and social. Empirical observations may not count as evidence in scientific settings but they may be considered evidence in statistical investigations carried out by young students (Makar & Rubin, 2009). According to the third important feature of ISI, an inferential claim should entail a level of uncertainty about the conclusions drawn. That is, students, using their own available language, should remain skeptical of conclusions made about unknown quantities based on known samples (Makar & Rubin, 2018).

The reasoning process that leads to an ISI is characterized as informal inferential reasoning (IIR). Zieffler, Garfield, delMas, and Reading (2008), after a scholastic review of the existing literature, provided a working definition of IIR as the "way in which students use their informal statistical knowledge to make arguments to support inferences about unknown populations based on observed samples" (pp. 44). This kind of reasoning incorporates the following elements (Ben-Zvi et al., 2007;

Ben-Zvi et al., 2012; Zieffler et al., 2008): (a) an evidence-based reasoning that leads to predictions and generalizations about a wider universe; (b) drawing on, utilizing, and integrating prior knowledge available (reasoning about variability, distributional reasoning, reasoning about signal and noise, sample reasoning, contextual reasoning, graph comprehension, reasoning about comparing groups, probabilistic reasoning, and inferential reasoning); and (c) articulations of uncertainty by using probabilistic language or references to the strengths and limitations of the inferred conclusions.

Ben-Zvi et al. (2007) examined primary school students' IIR in grades 4–6 through a longitudinal study grounded in social constructivist principles, using the Connections Learning Environment as the framework for their research. The IIR environment developed in the context of the project combined cognitive as well as sociocultural aspects of reasoning about data and chance. Students were not directly taught statistical concepts and procedures but were exposed to them through investigative classroom work. The cognitive aspects of IIR included several forms of statistics-related reasoning, such as the examination of patterns to form generalizations, the presence of variability, and the need for data (samples) to articulate conclusions. Sociocultural aspects included implicit rules such as norms prevailing in certain situations (that is, mathematical or social facts and operations established within a discourse), factors of task design that could support inference, and personal dispositions and experiences. We employed the framework of Ben-Zvi and colleagues to examine the multiple perspectives of students' emerging IIR. However, because our participants were second-grade students and thus were completely unfamiliar with (formal) statistical concepts and procedures, we reformulated the framework to serve the needs of our study. (See Section 3.2 for details.) By differentiating between cognitive and sociocultural aspects of reasoning, we do not assume that these aspects exist separately from one another. On the contrary, we presume that cognitive and sociocultural aspects are interwoven when a student attempts to reason inferentially.

Before proceeding, we should clarify that throughout this article, we refer to the reasoning of young students with no prior (formal) statistical experience. Hence, we did not anticipate students reasoning explicitly with concepts such as variability or sample size that were mentioned in the framework by Ben-Zvi et al. (2007). Makar (2018) introduced the theoretical notion of statistical-context structures in order to overcome the difficulty of referring to formalized concepts while studying young students' reasoning with powerful statistical ideas. Statistical context structures characterize problem contexts that can expose children to key statistical ideas. Statistical context structures are seen as "a mapping between a connected web of statistical structures (concepts with their related characteristics, representations, and processes) and contextual entities that stand in for the statistical structures" (pp. 7–8). For example, the typical shoe size of children in a classroom is a conceptual entity that could enable students to reason about the concept of central tendency without referring explicitly to the concept of statistical mean. In our study, students reasoned about contexts and approached powerful experiences with informal versions of statistical structures presented in the cognitive aspects of IIR. For example, they did not directly refer to the notion of sample while interacting with data but needed it to investigate the main question of the task. As a result, a contextual entity—a text written in any kind of book—would stand for the concept of sample for these second-grade students.

2.2. INFERENTIALISM AND IIR

Inferentialism is a semantic theory rooted in pragmatics that has been increasingly employed in statistics education in order to view old research themes under a new lens that highlights normative aspects of learning. According to its main founder, philosopher Robert Brandom (2000, as cited in Makar & Rubin, 2018), inferences—i.e., logical connections between concepts—are placed at the heart of human knowledge. Concepts attain their meaning through their use in reasoning (Bakker & Hußmann, 2017). Due to the focus this theory places on human reasoning, inferentialism has been useful while studying reasoning processes (Schindler & Seidouvy, 2019).

Before exploring inferentialist ideas about the social nature of reasoning, it is crucial to elaborate the inferentialist concepts of *commitment* and *entitlement*. As Hußmann and colleagues (2018) pointed out, inferentialism views commitments "as assertions that a person avows which are propositionally contentful and stated within discursive practices" (p. 136). Commitments refer to a person's perception of the world and constitute discursive practices because they reflect the norms to which a person adheres. Commitments can be assertional, arising when an individual makes an assertion or judgment about a proposition, or inferential, when the assertions are based on inferential relations between propositions. Individuals make their commitments explicit every time they are asked to reason. Commitments represent the smallest units of thought and action that individuals acknowledge within themselves and attribute to other individuals. Entitlements emerge from the specific commitments individuals make within discourse. After individuals recognize the implications of their commitments in employing a concept, they can discern what entitlements they have as a consequence of these commitments. The concept of entitlement refers to the capacity of a commitment to serve as a premise or justification for other claims. In inferentialist theory, commitments and entitlements are conceptualized as normative or deontic statuses.

In inferentialism, reasoning is a "social process of exteriorized linguistic moves," which can be described in terms of a "social assessment" (Noorloos et al., 2017, p. 447). More specifically, reasoning is a social process because students connect concepts and form commitments that are being tested through interactions with others. By these means, they extend or modify their understanding of the concepts they use because the rigor of their claims develops according to how their commitments are evaluated by others. Students' reasoning is social even when they are working alone because it has social origins and reflects social situations and norms. The idea of "exteriorized linguistic moves" is connected with the Game of Giving and Asking for Reasons (GoGAR). This game is connected to Wittgenstein's (1953) language game and is a practice in which participants provide utterances and take actions shaped by societal factors, including authority relations, duties, and responsibilities. The term "game" is used to highlight the importance of making moves, such as utterances, within the linguistic practice (Schindler & Seidouvy, 2019).

Although Wittgenstein rejected the idea of a central language game fundamental to all others, Brandom (2000, as cited in Noorloos et al., 2017) argued for it, identifying it as the practice of reason-giving through making assertions. For Brandom, GoGAR was a discursive practice in which persons exchange claims to which they commit themselves. It typically entails interactive participation between at least two speakers. Each time one speaker makes a claim, the responsibility falls upon the other to evaluate whether or not that claim is justified (Taylor et al., 2017).

GoGAR is closely connected to "deontic scorekeeping," an inferentialist concept that captures the discursive activity of monitoring one's own and others' assertions. This concept was explained by Noorloos et al. (2017) through the following example. If, during a discussion, a student asserts that "*p* is an equilateral triangle" and later claims that "*p* contains a 90-degree angle," it falls upon other students to request reasons supporting these assertions. Through deontic scorekeeping, students attend to the commitments that are being expressed inside a discursive practice and evaluate their correctness, appropriateness, and logical power (Derry, 2017). The cognitive activity that comes into play during this process is not stored in any individual's mind but is understood in terms of the norms within a particular discourse that imply permitted or forbidden moves in the language game. As Patel and Pfannkuch (2022) pointed out, the exchange of reasons between speakers prompts responses that may lead to agreement (sanction), disagreement (censure), or further questioning for clarification or alternative reasoning.

During deontic scorekeeping, individuals engage in the acknowledgment and attribution of commitments and entitlements to keep track of the scores in the game (Steiner, 2014). Acknowledging a commitment means being disposed to assert a particular sentence outright, especially when prompted. Regarding inferential commitments, acknowledging a commitment involves recognizing the inferential relations between sentences in the language fragment. Acknowledging an entitlement entails recognizing oneself as obliged or required to maintain a disposition in accordance with the norms governing discourse. By acknowledging commitments or entitlements, a player contemplates their personal beliefs (commitments) and what they take to have the permission, authorization, or authority to think or say (entitlements). Thus, acknowledging constitutes an intrapersonal activity. Attributing a commitment involves recognizing the assertional and inferential commitments of others within the context of discourse. While attributing an entitlement, a player assigns to the speaker (another player) the authorization to be committed to something. The attribution of a commitment or an entitlement is an interpersonal activity (Loeffler, 2021).

In inferentialism, learning is a matter of socio-cognitively arguing and participating (Taylor et al., 2017). Cognitive activity is not limited to objects stored in any individual's mind but is shaped by the individual's permitted or forbidden moves in the GoGAR. Students participating in a GoGAR mutually assess cognitive notions, and in this sense, the notions are being built up during the game itself. During this game, the notion of commitment is fundamental. The (inter)activity of participants in the GoGAR constitutes a "space of reasons," that is, a space in which logical commitments are made and justified as resulting from previously established commitments. During the course of both their cognitive and social development, students are introduced to the space of reasons. When students take part in the GoGAR, they navigate an established part of this space or explore a new area within it in an attempt to make effective moves in the game. So, inferentialism can possibly resolve the social–individual dichotomy by internally relating individual commitments to the social context if they are to be uttered or understood (Noorloos et al., 2017).

Based on the above theoretical considerations, inferentialism has been employed to investigate several issues in statistics and mathematics education settings. Schindler and colleagues (2017) investigated how primary school students relied on prior experiences while reasoning about negative numbers. The researchers concluded that inferentialism helped them to understand why students used specific ways to solve problems and why they used a certain way of thinking. Hußman et al. (2018) utilized inferentialism in order to understand the process of secondary students' conceptual development about decimal numbers. They argued that conceptual meaning could be understood by focusing on language games, in particular on students' reasoning in such games, the way students used the concepts, and the way that assertions served as reasons in such discursive practices. Seidouvy and Schindler (2019) examined both the individual and social aspects of primary school students' collaboration, paying particular attention to how students used norms, made commitments, and engaged in deontic scorekeeping. Their results illustrated the interplay between individual commitments and group dynamics in reaching a shared understanding, highlighting the importance of making inferential relations explicit in collaborative problem-solving.

Similar to our study, Schindler and Seidouvy (2019) examined the social nature of secondary school students' informal inferences. They discovered that students took different roles in moving IIR forward while socially negotiating the meanings of statistical concepts and that normative aspects appeared to influence students' IIR development. Incorporating inferentialist views into IIR, the authors advocated that students' discussions emerging during a statistical inquiry could be viewed as a GoGAR. In this game, students would make statements, give reasons, ask for reasons, acknowledge reasons, attribute these reasons to themselves or others, or abandon them. The term "game" highlights the importance of specific, even implicit, rules about how to make moves in the game, how to articulate claims within the classroom, or how, when, and for whom to ask reasons. Through the inferentialist lens, IIR is viewed not only statistically but also socially. Given that IIR takes place in social situations inside or outside the mathematics classroom, it unavoidably reflects the norms implanted in these situations.

The study reported in this paper investigates the interaction between cognitive and sociocultural aspects of students' reasoning in order to capture their IIR as a social practice traced in their dialogic talk. For this investigation, we sought theories that described reasoning and learning as products of social activity and emphasized the role of language in meaning-making and knowledge construction. Inferentialism offered a dynamic view of students' reasoning by focusing on the role of language and social practices in the construction of meaning and knowledge through the critical notions of commitments, entitlements, and deontic scorekeeping. These notions allowed description of students' reasoning and emphasized language not only as a tool for communication but also as a system of norms governing the use of concepts.

In our attempt to investigate students' IIR using inferentialist notions, we are aware of some difficulties that rest in the theoretical perspective employed, as identified in existing literature discussing inferentialist theory (e.g., Radford, 2017). Firstly, what students express through language is a thin layer of what they know and are aware of that cannot be fully described through their participation in language games. Secondly, in such games, students are perceived as rationally making their reasoning explicit, which is often difficult due to affective factors (Radford, 2017). Acknowledging Radford's remarks, in this study we employed the theoretical lens of inferentialism to examine students' IIR, focusing on the following research questions:

1. Which cognitive and sociocultural aspects of informal inferential reasoning do students incorporate in their commitments while engaging in a carefully designed inferential task?
2. In which way are the aforementioned aspects related within the game of giving and asking for reasons?

3. METHODS

For the purposes of the larger study from which the study in this paper emerged, a design experiment approach was adopted given the need to examine students' achievements in a specific mathematical topic (statistical inference) and inside a learning ecology (Cobb et al., 2003), that is, inside a complex, interacting system involving multiple cognitive and social elements. The preparation and design phase of the larger study lasted for over three months. During this period and based on research findings regarding how elementary school students' IIR evolves, the researchers recorded and systematically grouped the effective strategies and practices identified in the literature, aiming to incorporate them into the main study. During the main phase of the study, data collection and analysis were carried out in parallel to allow for demanding tasks to be alternated. Additionally, multiple data sources (recordings of the students' discussions, answers and notes on worksheets, or comments from the observation of students' work by the teacher-researcher) were utilized to ensure that sufficient answers to the research questions could be achieved. Because the study focused on young students' discussions in GoGAR contexts, the concern was to ensure that all students expressed their commitments, and these were evaluated by the other members of the group according to the process of deontic scorekeeping and not by simply accepting or rejecting them. To achieve this, short questions were posed by the first author, who also served as the class teacher, to either individual students or groups, aiming to stimulate a productive exchange of ideas and reasoning on issues identified in a previous phase of data collection and analysis as being inadequately addressed.

To clarify methodological and practical issues that emerged during the design of the research, a pilot study was conducted. Our objective was to test the suitability of the tasks for students' age and comprehension level, to test the data production processes, and to develop and primarily evaluate initial data analysis schemes. The pilot study led us to reflect on the positive aspects of each task as well as the concerns arising from task implementation, leading to revisions in the design of the main study. In terms of the data analysis, the concept of deontic scorekeeping appeared to be useful in determining how students adjusted the reasoning embedded in their claims based on the position these claims took in the group discussion and the evaluations from their peers. Following the pilot study, which lasted for over one month, the main study was designed during the 2020 spring semester and implemented from May 2021 until June 2021. The data examined in this paper pertain to a single group of students who participated in the main study.

During group discussions, the first author/teacher involved in the study acted as a participant observer. That is, she observed the group's discussion without commenting on the ideas expressed but could ask students to explain or clarify aspects of their activity and reasoning, paying attention to their interaction and communication. Although the teacher's role in the development of students' IIR can be highly significant, the investigation of student and teacher interactions while engaging in inferential activities is beyond the scope of this article. The data used in the study came from a class with 20 grade 2 students (age 8 years) in an urban public primary school in Greece during the 2021 spring semester. The students, 11 boys and nine girls, came from diverse socioeconomic backgrounds. The students were divided into groups of five, ensuring that each group included two members with strong communication skills. The rest of the students in each group were selected based on the working groups formed during the school year when students were required to discuss a topic among themselves. In this article, the work of a group of five students is described: Phroso, Ariadne, Grigoris, Giannos, and Phevos (all names are pseudonyms). This group caught our attention because of its members' abilities to verbally communicate their ideas and the rich discussions in which they engaged. The students' profiles are described based on the observations of the first author, who taught the class for two consecutive years. Phroso and Ariadne were two students who exhibited high verbal and communication skills and tended to seek deeper meaning in the knowledge they attained. They could be characterized as high-achieving students in mathematics. Grigoris was a student who showed great enthusiasm for tasks that he found interesting, paying attention to details. He was also interested in mathematics and was confident about his mathematical competence. Phevos and Giannos were students who typically gave short, one-word answers and often required encouragement to share their opinions. They appeared reluctant toward mathematics, often seeking verification from the teacher or their classmates while working on mathematical tasks, likely due to a lack of confidence in their mathematical abilities.

3.1. TASK DESIGN AND CLASSROOM NORMS OF COMMUNICATION

The main task of the study described in this paper was inspired by a children's book entitled *Martha Blah Blah* (Meaddaugh, 1994). Martha is a dog able to speak human language because her meal consists of a soup containing letters of the English (Greek, for the needs of our study) alphabet. Unfortunately, the company that produces her food is forced to eliminate some letters from the soup due to financial reasons. Utilizing this plot, we asked students to decide, as if they were the company's directors, which letters should be eliminated from the soup so that the company reduced costs and Martha retained a comprehensible level of speaking.

We used the task in a group work setting, aiming to encourage students to socially negotiate different aspects of the data. The class dealt with the task during some thematic workshops, which form part of the school curriculum in Greece to develop students' critical and thinking skills. At first, students participated in skill-building activities that involved making predictions based on a storybook, engaging in a small-scale statistical inquiry, and making predictions about larger samples. Then, students were introduced to the main task of the study, which was structured into six phases (see Table 1). Students were introduced to growing samples of 10, 50, and 100 letters and were invited to reconsider the main question and decide whether to change their decision in light of data from a larger sample. In the final phase of the task, students were encouraged to use Tinkerplots software (Konold & Miller, 2011) in order to investigate sampling distributions of larger or smaller sample sizes. These distributions were produced by the Tinkerplots sampler based on the assigned relative frequency of each letter as determined by previous quantitative linguistic studies.

Table 1. Overview of phases

Phase	Duration	Description
1	45'	Introduction to the task and 'brain-storming' inquiries without collecting data.
2	60'	2.1. Group discussion regarding data collection strategies and determining an appropriate sample size. 2.2. Making a decision after performing a small-scale data collection.
3	45'	Investigating two samples of 10 letters generated by Tinkerplots, deciding which letters to exclude, and forming hypotheses for larger samples of 50 letters.
4	45'	Investigating two samples of 50 letters generated by Tinkerplots, deciding which letters to exclude, and forming hypotheses for larger samples of 100 letters.
5	60'	5.1. Investigating two samples of 100 letters generated by Tinkerplots, deciding which letters to exclude, and forming hypotheses for larger samples of 500 letters. 5.2. Comparing the distributions of different sample sizes to identify patterns and variations.
6	60'	6.1. Observing and commenting on distribution changes with increasing sample sizes while working with the Tinkerplots software. 6.2. Observing and comparing different distributions of a specific sample size in the Tinkerplots software.

The design of the mathematical activity enacted by the task was guided by principles of mathematical inquiry. Students were invited to examine ill-structured problems that were based on mathematical (or statistical) evidence (Fielding-Wells & Makar, 2015). Answering the task's main question required students to negotiate the meaning of ambiguous phrases (comprehensible level of speaking), the solution plan (how can we know for sure that a certain letter is not frequent in Greek?), and the evaluation criteria concerning their response (how many letters should be eliminated?).

The present study investigated students' IIR as they worked on a sampling distribution—a concept that could provoke rich statistical discussions, given that it is central to a web of interconnected statistical ideas (Garfield & Ben-Zvi, 2009). To provide as many opportunities as possible for IIR to emerge, we supplemented the main task with sub-tasks. One of these sub-tasks employed the growing samples heuristic (Konold & Pollatsek, 2002), which required students to make sense of a data set and formulate an informal inference. Students were then asked to predict what would remain the same and what would change in a larger sample (Ben-Zvi et al., 2012). This heuristic could allow students to reflect upon stable features of distributions and compare predictions and speculations with actual data (Manor Braham & Ben-Zvi, 2015). Also, the main task required comparisons of two distributions representing data from different sample sizes (10, 50, 120, and 500 letters) and from different samples (different texts). Konold and Pollatsek (2002) claimed that the comparison of distributions enables students, both novices and experts in statistics, to examine not only measures of central tendency but also features such as variation and shape.

Given that the students of our study were expected to participate in a GoGAR in which social interactions are important, special attention was paid to acquainting students with how to engage in rich discussions, negotiate meanings, question each other, and defend their arguments. Throughout the school year, students were provided with numerous opportunities to engage in routines useful for promoting an effective classroom climate for that cause (Garfield & Ben-Zvi, 2009). Thus, they were asked questions with no singular correct answer and were encouraged to explain their reasoning, justify their answers, and make conjectures. The first author/teacher often asked students to comment on other students' conjectures and encouraged them to test their conjectures if the appropriate means were available. Furthermore, students were encouraged to engage in group discussions and to make decisions while respecting all expressed views.

3.2. DATA COLLECTION AND ANALYSIS

In order to determine the aspects of IIR and their relations in GoGAR, thematic analysis techniques were employed to analyze the data. The authors transcribed data from the group's audio discussions (data produced during the fourth phase of the task—see Table 1) and located excerpts that corresponded to the research questions. Then each independently identified the students' commitments (Hußman et al., 2018) in these excerpts. (The second author is a mathematics educator and researcher.) Commitments were reconstructed in a way that their content was meaningfully close to the initial articulation (Schacht & Huβmann, 2015) in order to deal with commitments productively for succeeding data analysis steps. In cases where students referred to statistical concepts using a statistical concept-structure (Makar, 2018), the authors reformulated the commitment using the statistical concept without implying that students possessed a formal understanding of the concept. After each researcher had identified and restated student commitments, discussions were held to ensure that commitments were characterized similarly, especially for the approximately 25% of cases where discrepancies in characterizations were identified. These latter commitments were reexamined within the contexts that emerged in the data, seeking evidence to support the characterization that best aligned with the respective student's argumentation within the group.

Having identified students' commitments, the researchers moved on to coding the cognitive and sociocultural aspects of IIR using the scheme suggested by Ben-Zvi and colleagues (2007) as a guide. Because some of the categories suggested by this scheme were either subdivided or were not identified in the data, new codes had to be introduced. The categorization by Ben-Zvi et al. (2007) also had to be expanded to incorporate aspects that occurred in the data and were relevant to IIR's development and not its outcome. We used the cognitive aspect of the "deterministic viewpoint" to describe students' non-probabilistic statements in the early stages of IIR development. Tables 2 and 3 present the elements of the original framework of Ben-Zvi et al. (2007) as well as the revised version for the cognitive and sociocultural aspects of IIR, respectively. The codes assigned to each cognitive and sociocultural aspect of reasoning in the revised scheme appear in parentheses.

Having traced each member's commitments as propositions that individuals hold to be true, we aimed to identify relations between reasoning aspects embedded in those commitments according to their position and interpretation in the GoGAR. Every assertion each student (player) made during discussion was considered a move made at each stage of the game. Using the inferential vocabulary of acknowledging and attributing commitments and entitlements described in Section 2.2 on the theoretical background, the authors traced those moves in the group's discussion. Because acknowledging is intrapersonal, we assumed that when a player asserted a statement, he or she also acknowledged the corresponding entitlement to that statement. Table 4 summarizes how the scorekeeping moves of acknowledging and attributing commitments or entitlements were coded during the analysis of the students' discussion.

Table 2. Codes and descriptors of cognitive aspects of IIR

Category	Ben-Zvi et al. (2007)	Revised scheme	
		Subcategory	Description
Distributional reasoning	Aggregate views, pattern and trend, hypothesis and prediction, as well as local reasoning about individual cases, outliers	Partial detection of patterns (C1)	Stable features between two data sets are identified.
		Total detection of patterns (C2)	Stable features between multiple data sets are identified.
		Focusing on individual aspects of distribution (C3)	Focus is placed on single data values or features.
		Viewing data as aggregate (C4)	Data is described without mentioning specific distribution aspects.
Reasoning about signal	Center, measures, modal clumps, summary	Measures of center (C5)	The mode of the sample is referred to.
Sampling reasoning	Sample size, randomness, sampling variability and behavior, bias, representativeness	Importance and role of sample size (C6)	Sample size is referenced when commenting on data frequencies.
		Sampling variability (C7)	Features varying between distributions from different samples are identified.
		Sampling bias and representativeness (C8)	Characteristics of sampling that influence data distributions are identified.
Reasoning about variability	Spread, density, variability from a variety of sources	Acknowledge and interpret variability in data (C9)	Features of distributions that vary due to multiple causes are identified.
Reasoning about uncertainty	Uncertainty, random events, chance	Acknowledge and realize the uncertainty of conclusions or predictions (C10)	Probabilistic vocabulary is used in reaching conclusions or making predictions.
		Early notions of randomness and probability (C11)	The probability of an event is referred to in students' own language.
Deterministic viewpoint	Not included in the framework	Certainty about conclusions and predictions (C12)	Deterministic language is adopted in reaching conclusions or making predictions.

Table 3. Codes and descriptors of sociocultural aspects of IIR

Category	Ben-Zvi et al. (2007)	Revised scheme	
		Subcategory	Description
Instructional context	Learning environment design, teachers' and students' awareness of purposes and utility	Learning environment design (S1)	Reference is made to elements related to the task being worked on.
		Teachers' and students' awareness of purpose and utility (S2)	Elements of the task are utilized independently to reach certain goals.
Culture and history	Students' beliefs, disposition, prior knowledge and background	Students' beliefs (S3)	Personal convictions are incorporated into reasoning without further explanations being provided for their correctness.
		Prior knowledge or experiences relevant to problem context (S4)	Previous experiences related to grammatical or linguistic phenomena are referred to.
		Prior knowledge or experiences relevant to mathematics (S5)	Previous experiences related to mathematics are referred to.
		General knowledge or experiences (S6)	Previous experiences related to various sources such as everyday life or personal experience are referred to.
Socio-statistical norms	Classroom discourse norms, what a statistical claim is, what it takes to be convinced that a claim is true or false	Norms prevailing classroom discussions (S7)	Implicit rules governing group discussions are evident in the statements.
		School life norms (S8)	Implicit rules for classroom behavior are identifiable in the statements.
		Mathematical norms (S9)	Implicit rules stemming from school mathematics teaching are detected in the statements.
		Emerging statistical norms (S10)	Implicit rules related to the experiences students gain in relation to statistics are detected in the statements.
Evaluative disposition	Providing and assessing evidence, level of confidence, critical disposition to sampling and inference	Category not included because analytical tools from inferentialism were used to specify evaluative dispositions in greater detail.	
Language	Discourse types and norms to discuss data, graphs, sampling, inferences	Category not included because of overlapping significantly with the "socio-statistical" norms category.	
Argumentation	Arguing about inferences, claims and counter-claims data-based evidence	Category not included because describing reasoning as argumentation was beyond the scope of the study.	
Flexibility	Transfer back and forth between local and global view of data, sample and population, data and context, reality and its representations	Category not included because it was not identified in the data.	

Table 4. Coding of scorekeeping moves

Scorekeeping move		Description
Acknowledge (intrapersonal)	a commitment	Assert a sentence.
	an entitlement	Recognize oneself as required to maintain a disposition in accordance with the norms governing the discourse.
Attribute (interpersonal)	a commitment	Recognize commitments *of others* within the context of the discourse.
	an entitlement	Assign another player (speaker) the authorization to be committed to something.

Through the acknowledgment and the attribution of commitments and entitlements between the players during the discussion, permitted or forbidden moves in the discourse were identified. A move was considered permitted (P) when a player acknowledged a commitment and the corresponding entitlement for himself, and another player responsively attributed this entitlement to him using words that expressed sanctioning (Patel & Pfannkuch, 2022) such as "yes" or "I agree" and/or by forming a new commitment that gave reasons about the approval of the first move. Furthermore, a move was considered permitted when a player acknowledged a commitment and an entitlement for himself, and another player responsively acknowledged the same commitment. The reasoning aspects embedded in commitments that indicated permitted moves in the discourse were labeled to share a corroborative relation (C.R.). A move was considered forbidden (F) when one player acknowledged a commitment and the corresponding entitlement for himself, and another player did not attribute the corresponding entitlement to him by using words that express censuring (Patel & Pfannkuch, 2022) such as "no" or "I don't think/know" and/or by forming a new commitment giving reasons for this criticism. The reasoning aspects embedded in commitments that indicated forbidden moves in the discourse were labeled to share a dismissive relation (D.R).

4. RESULTS

In this study we focused on the students' group discussion after examining two samples of 50 letters. Beforehand, the group had already participated in three phases of the task implementation, as described in Table 1. Table 5 displays the aspects of reasoning that emerged during those previous phases.

Table 5. Reasoning aspects that emerged from previous phases of the group's engagement with the task

	Absence of data	Small-scale data collection	Data from two samples of 10 letters
Cognitive aspects	Not identified	Not identified	• Partial pattern detection (C1) • Focus on individual aspects of distribution (C3)
Sociocultural aspects	• Previous experiences related to problem context (e.g., linguistic features of Greek language - homophony) (S4) • Students' personal beliefs (S3) • Learning environment design (S1)	• Previous experiences related to problem context (e.g., linguistic features of Greek language-homophony) (S4) • Students' personal beliefs (S3) • Learning environment design (S1) • Purpose and utility (S2)	• Previous experiences related to problem context (linguistic features of Greek language-homophony) (S4) • Students' personal beliefs (S3) • Learning environment design (S1) • Purpose and utility (S2) • Mathematical norms (comparison of frequencies with ten) (S9)
Aspects and certainty state	Complete certainty (C11), Norms of group discussion (S7)		

During these previous phases of the task, the members of the group drew upon cognitive and sociocultural aspects of reasoning to determine which letters should be eliminated from the dog's soup. In all three phases, they remained completely certain about their decision because it was a product of group discussion in which each member's opinion was respected and considered ("*We listened to everybody's opinion and came to an agreement"*). It is important to note that this view on group discussions has been cultivated by the teacher since the beginning of the school year as part of this classroom's discourse. We decided to focus on the group's decision while examining samples of 50 letters because this phase was a turning point in the group's certainty state in that members provided evidence of uncertainty about their decision for the first time during this phase of the group discussions.

The following dialogue between the members of the group took place after they had commented on data from two samples of 50 letters and were about to decide which letters should be eliminated from the dog's soup. The diagrams in Figures 1 and 2 display the data the group examined during the discussion below. Analyses of the dialogue are provided in Sections 4.1 and 4.2.

1	Ariadne:	Let's leave out "zeta," it has not appeared in any of the texts we have.
2	Grigoris:	Yes! I say we should eliminate "omega" because we have "omicron" as well.
3	Giannos:	I say we should omit "chi," "psi," and "delta"… those [letters] that don't appear in either of the texts.
4	Phevos:	Why don't we eliminate some letters that appear one or two times as well? [For example] "rho," "phi"…
5	Phroso:	We could leave out "beta" and "theta," that appear only once.
6	Grigoris:	Well, if we eliminate those as well … Previously [while examining a sample of 10 letters] we eliminated 10 letters and we shouldn't have; Martha would be unable to speak! I think we should eliminate 1, 2, or 3 letters.
7	Phroso:	I agree, we eliminated too many letters … If we worked in the factory, we would like to reduce our costs, and this [eliminating 10 letters] would be beneficial. But we saw that other groups eliminated a small number of letters, so we can do so as well.
8	Ariadne:	I think that the tables we have helped us more to decide today. We saw that some of the letters we eliminated yesterday appeared many times. I think we should eliminate "delta," "zeta," and "omega."
9	Phroso:	I agree, "delta" and "zeta" do not appear even once and "omega" can be replaced by "omicron."
10	Grigoris:	Yes, we can eliminate those … If we eliminate "psi" and "chi," we will eliminate 5 letters in total, I think 5 letters are too many letters to be missing.
11	Phroso:	I think that when we count 100 letters, we will change our decision. Because in 100 letters, many letters will appear more times than they appear in 50 … I mean, if one letter doesn't appear even once in 50 letters, in 100 letters it could appear let's say 11 times. That's why I am not completely sure that those letters we suggest right now are the suitable ones to take out of the soup.
12	Grigoris:	I agree with you, in 100 letters we won't have the same times as we have here. The times each letter appears will change, as they did today.
13	Giannos:	I am not completely sure either. I would change my decision if I counted 100 letters, because my previous decision was completely wrong.
14	Phevos:	Me too … given that the letters I suggested previously were wrong, the letters I suggest now might be wrong as well.
15	Ariadne:	Well, I don't think I would change our decision. I think those letters we suggested now are okay, we can see that they don't appear many times. In 100 letters, they could appear one or two times, but not many times.
16	Phroso:	Yes, but we are talking about 100 letters, many letters … I don't know if they will be appearing just a few times.
17	Giannos:	I think we should reply that we would change our opinion, counting 100 letters. Ariadne, most of us support this view.
18	Ariadne:	Yes, you are right. Let's reply this way.

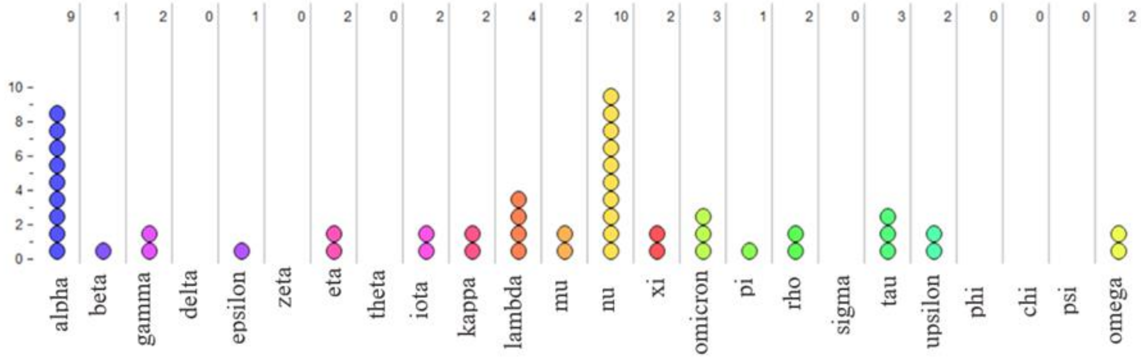

Figure 1. Letters and their frequencies from the first sample of 50 letters

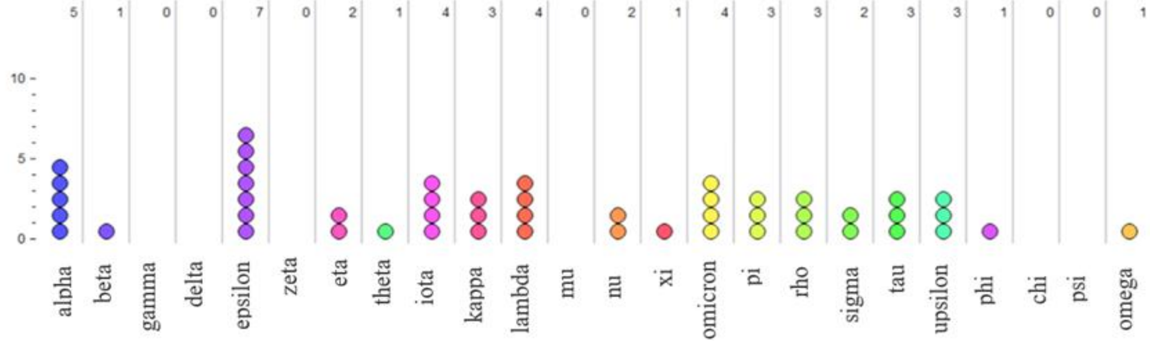

Figure 2. Letters and their frequencies from the second sample of 50 letters

4.1. STUDENTS' INDIVIDUAL COMMITMENTS AND REASONING ASPECTS

This section focuses on the cognitive and sociocultural aspects of IIR identified in the commitments expressed by each individual student member of the group under consideration in this study. Tables 6 through 10 present these aspects for each of the five students.

While engaging with the sample size of 50 letters, Phroso seems to focus on letters that she characterizes as having a small frequency (Line 5, Table 6). In the previous phase, she commented that letters appearing only once should be eliminated because "one is not much compared to 10." We view this kind of characterization as normative because, in Greek mathematics education, students often use 10 to execute arithmetical operations easily. Phroso also comments that during previous phases, the group eliminated too many letters based on the design of the task, but now this could change given the fact that other groups eliminated a small number of letters (Line 7). In this way, she integrates school norms into her commitments, placing emphasis on what other groups believe rather than the instructional design. We refer to school norms as social facts established within a discourse concerning how a student should work or behave in the classroom. It is crucial to note that in the Greek mathematics teaching tradition, ill-structured problems are not commonly used. As a result, students tend to think that there is a single correct answer to questions, so their work should align with the work of the rest of the class. As far as the group decision is concerned, she appears uncertain, noticing data variability as the sample size increases (Line 11).

Table 6. Phroso's aspects of IIR

Line	Statement	Commitment	Aspect of IIR
5	We could leave out "beta" and "theta," that appear only once.	Letters whose frequency is characterized as small can be eliminated.	• C.A. Individual features of a distribution (C3) • S.A. Mathematical Norms (S9)
7	I agree, we eliminated too many letters … if we worked in the factory, we would like to reduce our costs and this [eliminating 10 letters] would be beneficial. But we saw that other groups eliminated a small number of letters, so we can do so as well.	Answers given by other teams affect our decisions more than the instructional context.	• S.A. School life norms (S8) • S.A. Learning environment design (S1)
11	I think that when we count 100 letters, we will change our decision. Because in 100 letters, many letters will appear more times than they appear in 50 … I mean, if one letter doesn't appear even once in 50 letters, in 100 letters it could appear let's say 11 times. That's why I am not completely sure that those letters we suggest right now are the suitable ones to take out of the soup.	• I can make a decision with a degree of uncertainty. • The increase of the sample size will provoke changes in data.	• C.A. Uncertainty (C10) • C.A. Variability (C9)

As the excerpts in Table 7 suggest, Ariadne looks interested in letters that appear zero times in both sample distributions available (Line 1) and comments that this time the group focused more on the available data (Line 8). Her certainty about the validity of the group's decision is based on this focus on available data (Line 15), but her commitment seems to be refuted by the rest of the group due to variability in data and norms of group discussion.

Table 7. Ariadne's aspects of IIR

Line	Statement	Commitment	Aspect of IIR
1	Let's leave out "zeta," it has not appeared in any of the texts we have.	Letters appearing zero times in both tables available should be eliminated.	• C.A. Individual features of a distribution (C3) • C.A. Partial pattern detection (C1)
8	I think that the tables we have helped us more to decide today. We saw that some of the letters we eliminated yesterday appeared many times.	Data can help us decide which letters should be eliminated.	• S.A. Purpose and utility (S2)
15	Well, I don't think I would change our decision. I think those letters we suggested now are okay, we can see that they don't appear many times. In 100 letters, they could appear one or two times, but not many times.	• I can make a decision with certainty. • The sample gives us all the information about the population.	• C.A. Certainty (C12) • C.A. Representativeness (C8)

Grigoris appears to incorporate cognitive aspects into his commitments, such as previous experiences concerning the Greek language (Line 2, Table 8), as well as sociocultural aspects such as factors related to task design. He places emphasis on the fact that Martha's speech should be comprehensible and suggests the elimination of a small number of letters (Line 6). He seems uncertain about the group's decision, noticing variability in the data as the sample size increases (Line 12).

Table 8. Grigoris's aspects of IIR

Line	Statement	Commitment	Aspect of IIR
2	I say we should eliminate "omega," because we have "omicron" as well.	• In the Greek language there are letters with the same pronunciation. • The more letters we eliminate, the better for the company's costs.	• C.A. Previous experiences related to problem context. (S4) • S.A. Learning environment design (S1)
6	Well, if we eliminate those as well… previously [while examining a sample of 10 letters] we eliminated 10 letters and we shouldn't have, Martha would be unable to speak! I think we should eliminate 1,2, or 3 letters.	If we omit a large number of letters, Martha will not speak clearly.	• S.A. Learning environment design (S1)
12	I agree with you, in 100 letters we won't have the same times as we have here. The times each letter appears will change, as they did today.	The increase of the sample size will provoke changes in data.	• C.A. Uncertainty (C10) • C.A. Variability (C9)

Giannos seems to be interested in letters with zero frequency in the available data, detecting the letters that follow this pattern (Line 3, Table 9). He seems unsure about the final decision of what letters to eliminate due to the rejection of his previous decisions by the growing samples task design (Line 13). This is because he predicted that certain letters would appear a small number of times in a larger sample, and this prediction was disproved by the actual data from larger samples presented by task design. He seems skeptical about Ariadne's certainty and tries to change her mind, drawing upon the norms of group discussion (Line 17).

Table 9. Giannos's aspects of IIR

Line	Statement	Commitment	Aspect of IIR
3	I say we should omit "chi," "psi," and "delta." Those [letters] that don't appear in either of the texts.	Letters appearing zero times in both tables available should be eliminated.	• C.A. Individual features of a distribution (C3) • C.A. Partial pattern detection (C1)
13	I am not completely sure either. I would change my decision if I counted 100 letters, because my previous decision was completely wrong.	• I can make a decision under uncertainty. • If my decision is proved wrong a number of times, I will change it.	• C.A. Uncertainty (C10) • S.A. Learning environment design (S1)
17	I think we should reply that we would change our opinion, counting 100 letters. Ariadne, most of us support this view.	While working in groups, the final decision should be made based on the opinion of the majority.	• S.A. Norms of group discussion (S7)

Phevos appears to be interested in eliminating letters appearing one or two times in the available data (Line 4, Table 10), but his commitment is refuted by Grigoris due to factors of instructional design. He appears uncertain about his decision because his previous decisions were proved wrong by the growing samples task design (Line 14).

Table 10. Phevos's aspects of IIR

Line	Statement	Commitment	Aspect of IIR
4	Why don't we eliminate some letters that appear one or two times as well? [For example] "rho," "phi"…	Letters whose frequency is characterized as small can be eliminated.	• C.A. Individual features of a distribution (C3) • S.A. Mathematical norms (S9)
14	Me too … given that the letters I suggested previously were wrong, the letters I suggest now might be wrong as well.	• I can make a decision under uncertainty. • If my decision is proved wrong a number of times, I will change it.	• C.A. Uncertainty (C10) • S.A. Learning environment design (S1)

Overall, while the students in this group attempted to make inferences from samples, they appeared to incorporate both cognitive and sociocultural aspects into their reasoning. In addition to cognitive elements related to the inference process, such as individual cases of the given distribution, pattern detection, variability, and uncertainty, the students also integrated elements related to their personal experiences or knowledge as well as the context in which they were operating—namely, the classroom.

4.2. RELATIONS OF REASONING ASPECTS DURING GOGAR

Having traced the aspects of each student's IIR, we now focus on the relations between these aspects inside the group's GoGAR. Players are indicated with the abbreviation of their name and a number indicating their turn during GoGAR (e.g., A1 to indicate Ariadne's first turn). The commitments (identified in the previous phase) are now indicated with a number according to the dialogue's sequence. If two players share the same commitment (Cmt), the same number is used to label the commitment for both players. Inferential commitments are denoted by an accent (') after the commitment number (e.g., Cmt7'). For the entitlements each player acknowledges for himself, the term "default entitlement" is employed to indicate that this entitlement (E) has not yet been attributed from another player but solely from the speaker himself. Table 11 delineates the scorekeeping moves (P indicates a permitted move, whereas F indicates a forbidden one) and corresponding relations (C.R. denotes a corroborative relation, whereas D.R. denotes a dismissive one) between reasoning aspects in each turn of the game.

According to Table 11, at the beginning of the dialogue, Ariadne (A) acknowledges the commitment and the corresponding entitlement that the letters appearing zero times in the available data should be eliminated (Line 1). Grigoris (Gr) attributes the corresponding entitlement to her and adds one more letter for elimination based on his previous experiences regarding the sound of Greek letters (Line 2). Giannos (Gi) appears to acknowledge the same commitment as Ariadne, which indicates the exclusion of letters that follow the zero-frequency pattern (Line 3). Phevos (Phe) suggests that letters whose frequency is considered low according to mathematical norms could be omitted, too, and Phroso (Phr) seems to acknowledge this commitment for herself as she proposes letters for elimination according to this criterion (Lines 4–5). Grigoris marks those two moves by Phevos and Phroso as forbidden inside the group's current discourse based on the instructional context and the design of the learning environment, which do not favor the elimination of a large number of letters (Line 6). This commitment becomes immediately acknowledgeable to Phroso, who enhances its validity based on school-life norms (Line 7). Ariadne acknowledges that during this phase of the task, the group focused more on the available data (Line 8). Phroso adds a commitment with which she seems to acknowledge the commitments of other players as her own, namely the partial pattern detection expressed by Ariadne and the previous experience about the sound of Greek letters expressed by Grigoris (Line 9). Grigoris attributes this entitlement to Phroso, commenting that it is also consistent with the design of the learning environment (Line 10).

Table 11. Relations among reasoning aspects in the GoGAR

Line	Player & Assertion	Commitment	Scorekeeping moves	Social evaluation of move & Corresponding aspect relations
1	A1: Let's leave out "zeta," it has not appeared in any of the texts we have.	Cmt1. Letters appearing zero times in both tables available should be eliminated.	Acknowledges assertional Cmt1 & default E1	P: detect patterns to reach a conclusion
2	Gr1: *Yes!* I say we should eliminate "omega," because we have "omicron" as well.	Cmt2. i) In the Greek language there are letters with the same pronunciation. ii) The more letters we eliminate, the better for the company's costs.	Attributes E1 to A1 Acknowledges assertional Cmt2 & default E2	C.R.: C1, C3, S4
3	Gi1: I say we should omit "chi," "psi," and "delta"… those [letters] that don't appear in either of the texts.	Cmt1. Letters appearing zero times in both tables available should be eliminated.	Acknowledges assertional Cmt1 for himself & default E1	
4	Phe1: Why don't we eliminate some letters that appear one or two times as well? [For example] "rho," "phi"…	Cmt3. Letters whose frequency is characterized as small can be eliminated.	Acknowledges assertional Cmt3 & default E3	P: use mathematical norms to characterize frequencies
5	Phr1: We could leave out "beta" and "theta," that appear only once.	Cmt3. Letters whose frequency is characterized as small can be eliminated.	Attributes E3 to Phe1 Acknowledges Cmt3 & default E3 for herself	C.R.: C3, S9
6	Gr2: Well, if we eliminate those as well… Previously [while examining a sample of 10 letters] we eliminated 10 letters and we shouldn't have, Martha would be unable to speak! I think we should eliminate 1, 2, or 3 letters.	Cmt4. If we omit a large number of letters, Martha will not speak clearly.	Acknowledges assertional Cmt4 & default E4 Does not attribute E3 to Phe1 and Phr1	F: use mathematical norms to characterize frequencies, Reason: instructional design D.R.: S1, C3, S9
7	Phr2: *I agree*, we eliminated too many letters… If we worked in the factory, we would like to reduce our costs, and this [eliminating 10 letters] would be beneficial. But we saw that other groups eliminated a small number of letters, so we can do so as well.	Cmt5. Answers given by other teams affect our decisions more than the instructional context.	Attributes E4 to Gr2 Acknowledges assertional Cmt5 & default E5	C.R.: S1, S8

Line	Player & Assertion	Commitment	Scorekeeping moves	Social evaluation of move & Corresponding aspect relations
8	A2: I think that the tables we have helped us more to decide today. We saw that some of the letters we eliminated yesterday appeared many times. I think we should eliminate "delta," "zeta," *and "omega."*	Cmt6. Data can help us decide which letters should be eliminated.	Attributes E2 to Gr1 Acknowledges assertional Cmt6 & default E6	P: Make use of the available data and the instructional design to reach a conclusion C.R.: S2, C1, S4
9	Phr3: *I agree*, "delta" and "zeta" do not appear even once and "omega" can be replaced by "omicron."	Cmt1. Letters appearing zero times in both tables available should be eliminated. Cmt2. i) In the Greek language there are letters with the same pronunciation. ii)The more letters we eliminate, the better for the company's costs.	Attributes E1 and E2 to A1 and Gr1 Acknowledges Cmt1, Cmt2, & default E1, E2 for herself	
10	Gr3: *Yes*, we can eliminate those … if we eliminate "psi" and "chi," we will eliminate 5 letters in total, I think 5 letters are too many letters to be missing.	Cmt4'. We cannot eliminate 5 letters, because 5 is a large number of letters to be eliminated.	Acknowledges inferential Cmt4' & default E4'	
11	Phr4: I think that when we count 100 letters, we will change our decision. Because in 100 letters, many letters will appear more times than they appear in 50… I mean, if one letter doesn't appear even once in 50 letters, in 100 letters it could appear let's say 11 times. That's why I am not completely sure that those letters we suggest right now are the suitable ones to take out of the soup.	Cmt7. I can make a decision with a degree of uncertainty. The increase of the sample size will provoke changes in data.	Acknowledges assertional Cmt7 & default E7	P: acknowledge variability while deciding the level of certainty in a conclusion C.R.: C9, C10
12	Gr1: I agree with you, in 100 letters we won't have the same times as we have here. The times each letter appears will change, as they did today.	Cmt7. The increase of the sample size will provoke changes in data.	Attributes E7 to Phr4 Acknowledges assertional Cmt7 & default E7 for himself	
13	Gi2: I am not completely sure either. I would change my decision if I counted 100 letters, because my previous decision was completely wrong.	Cmt8. I can make a decision under uncertainty. If my decision is proved wrong a number of times, I will change it.	Acknowledges assertional Cmt8 & default E8	P: acknowledge instructional design while deciding the level of certainty in a conclusion C.R.: S1, C10
14	Phe2: *Me too…* given that the letters I suggested previously were wrong, the letters I suggest now might be wrong as well.	Cmt8. I can make a decision under uncertainty. If my decision is proved wrong a number of times, I will change it.	Attributes E8 to Gi2 Acknowledges assertional Cmt8 & default E8 for himself	

Line	Player & Assertion	Commitment	Scorekeeping moves	Social evaluation of move & Corresponding aspect relations
15	A3: Well, *I don't think* I would change our decision. I think those letters we suggested now are okay, we can see that they don't appear many times. In 100 letters, they could appear one or two times, but not many times.	Cmt9. I can make a decision with certainty. The sample gives us all the information about the population.	Acknowledges assertional Cmt9 & default E9 Does not attribute E7, E8 to the rest of the group	F: Rely solely on sample representativeness while deciding the level of certainty in a conclusion Reason: Variability
16	Phr5: Yes, but we are talking about 100 letters, many letters … *I don't know* if they will be appearing just a few times.	Cmt7'. The increase of the sample size from 50 to 100 letters will provoke changes in data	Acknowledges assertional Cmt7' & default E7' Does not attribute E9 to A3	F: Rely solely on sample representativeness while deciding the level of certainty in a conclusion, Reason: norms of group discussion D.R.: C7, C8, C10, C12, S7 S1
17	Gi3: I think we should reply that we would change our opinion, counting 100 letters. Ariadne, most of us support this view.	Cmt10. While working in groups, the final decision should be made based on the opinion of the majority.	Acknowledges assertional Cmt10 & default E10 Does not attribute E9 to A3	
18	A4: *Yes, you are right*. Let's reply this way.	Cmt10. While working in groups, the final decision should be made based on the opinion of the majority.	Attributes E10 to Gi3 Acknowledges assertional Cmt10 & default E10 for herself	P: acknowledge the norms of group discussion while reaching a conclusion in a group C.R.: C10, S7

As for the final decision, Phroso acknowledges that the group should appear uncertain due to the variability in the data (Line 11). Grigoris attributes this entitlement to Phroso, also mentioning variability in data and acknowledging this commitment for himself (Line 12). Giannos and Phevos also approve of uncertainty by acknowledging commitments that referenced previous wrong decisions due to the growing samples task design (Lines 13–14). Only Ariadne acknowledges herself to be certain about the group's conclusion due to sample representativeness (Line 15). Phroso marks this move as forbidden by acknowledging an inferential commitment referring to variability (Line 16). Giannos acknowledges the commitment that all decisions should be taken by most students in the group (Line 17). That is when Ariadne acknowledges this commitment for herself and is thus persuaded to agree with the rest of the group (Line 18). By viewing group discussion as a GoGAR and identifying the acknowledgment and attribution of commitments and entitlements, it becomes evident that cognitive and sociocultural aspects do not stand in isolation.

Students related cognitive and sociocultural aspects of reasoning as they attempted to validate or reject statements during discussions. Often, cognitive and sociocultural aspects "worked together" for students to make sense of the data and attribute entitlements to other members of the group. In the group's dialogue, members of the group corroboratively related patterns identified in the data with their prior linguistic knowledge (e.g., Line 2). Additionally, students related elements of instructional design, such as the need of finding a cost-effective solution, with school life norms and the available data provided to them (e.g., Lines 6–9). Finally, they connected their uncertainty about conclusions both with data variability and with the rejection of previous hypotheses, influenced by the increasing sample strategy incorporated into the task's design (e.g., Lines 11–12).

However, there were also cases where cognitive and sociocultural aspects came into conflict and shared a dismissive relation. In cases of dismissive relations, students connected certain aspects of reasoning to withhold entitlement to specific commitments within the group's discourse. Given that using mathematical norms to classify frequencies as high or low led to the exclusion of a large number of letters, design elements of the learning environment were used to forbid their use in the final decision (Lines 5–6). Similarly, certainty due to sample representativeness was recognized by one group member but not accepted by the others due to sampling variability and discussion norms that emphasized respect for the majority (e.g., Lines 15–18).

These corroborative and dismissive relations are presented schematically in Figure 3. Abbreviations on the vertical axis refer to students within the group's interaction, with each represented by a different color. The cognitive and social aspects each student incorporated into his emerging IIR follow the codes appearing in Tables 2 and 3. Each green arrow stands for a corroborative connection within the aspects of reasoning. Each red arrow implies a dismissive connection between the reasoning aspects.

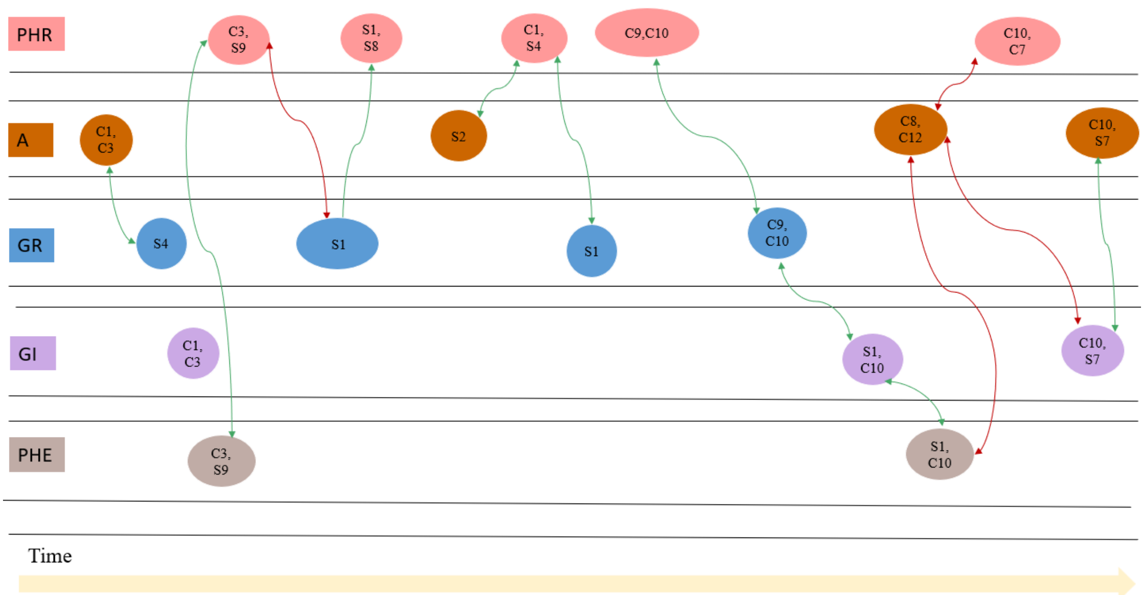

Figure 3. Schematic representation of corroborative and dismissive relations between aspects

5. DISCUSSION AND CONCLUSIONS

The purpose of the study reported here was to address different aspects of students' IIR. In particular, we examined the cognitive and sociocultural aspects emerging in young students' IIR as well as the relations between those aspects during students' group work. Overall, our findings seem to support the view of Schindler and Seidouvy (2019) that students' IIR is the outcome of a synergy of statistical content and social practice in which students engage in different ways. This is because, by examining students' IIR, we traced not only cognitive aspects about statistical concepts and procedures but also sociocultural aspects such as context or social interactions within the group. These cognitive and sociocultural aspects were often interwoven, either corroboratively or dismissively, while students attempted to reach conclusions.

Detecting patterns and trends in data is a crucial practice in statistics because it sets the basis for making (probabilistic) generalizations (Konold & Pollatsek, 2002). Given that students in our study were encouraged to generalize from data samples to a larger population, the cognitive aspect of pattern detection was more or less expected to emerge in their IIR. The participating students detected patterns about individual letters with frequencies of zero while comparing two sample distributions of equal size. Other cognitive aspects detected were variability and uncertainty—concepts that are crucial while reasoning inferentially (Ben-Zvi et al., 2012). But, as we studied students' emerging IIR, we also detected the cognitive aspect of certainty in students' reasoning without acknowledgment to the presence of uncertainty. When it comes to available data, students commented on individual aspects of these data while looking at sample distributions, similar to findings from other studies examining young students' IIR (Paparistodemou & Meletiou-Mavrotheris, 2017). Furthermore, some students in our study relied on sample representativeness while drawing conclusions.

As far as sociocultural aspects of reasoning are concerned, our students seemed to draw upon linguistic features of the Greek language related mainly to phonetics to eliminate letters with special characteristics (i.e., the same sound as others that are not eliminated). This kind of contextual knowledge and experience is, according to Makar et al. (2011), an element that can support students' IIR in the classroom. The sociocultural aspect of instructional design became apparent when students were commenting on available data to shape their conclusions. That is, they made use of the tables provided by the task to form commitments, whereas in the preliminary phases, they only focused on data that was compatible with their personal beliefs. Recognizing the need for data to answer a research question is a crucial step in early statistical thinking because it links the question with the evidence needed to check if a claim is true (Makar, 2018).

Normative aspects of reasoning, such as the need to have a single value as an answer or to draw on teacher authority, have been identified by studies examining middle school students' IIR (Schindler & Seidouvy, 2019). Our study enriches these findings by identifying aspects such as school life norms or group discussion norms emerging in students' IIR. School life norms entail students' perceptions about how someone should work in the classroom, specifically that if an answer does not align with the answer of the majority, it is wrong. Furthermore, students drew upon norms of group discussion in their attempt to reach a consensus about the group's certainty state. Apart from those circumstances that were relevant to making specific decisions, normative aspects also emerged in cases where students were dealing with data. While examining the sample of 50 letters, students tended to focus on the data, utilizing the available tables to reach a conclusion. Initially, they appeared to characterize frequencies in the data as large or small, following mathematical norms that might have been shaped by teaching and learning natural numbers in the first and second grades. This is because the participating students tended to characterize a number's value as small or big compared to ten, the base of the decimal system emphasized in mathematics teaching.

Our results indicate that the above-mentioned cognitive and sociocultural aspects share some corroborative and dismissive relations when used to form commitments during a GoGAR. To reveal these relations, we conceptualized students' discussion as a language game, where each student (player) produced assertions (commitments) that constituted moves in the game. These moves can be permitted or forbidden according to the rules or norms that currently prevail in the group's discourse. As far as corroborative relations are concerned, the students tended to closely relate the cognitive aspect of variability with that of uncertainty. In particular, students who were uncertain about the group's conclusion related the aspect of uncertainty with that of variability in order to gain the corresponding entitlement to be uncertain from the other players in the game. This connection was considered a permitted move in the group's discourse. Even though one player forbade this move using the aspect of sample representativeness as a reason, she retracted this commitment due to the norms of group discussion.

Players of the game corroboratively related the aspect of uncertainty with the sociocultural aspect of instructional design. It appears that students were committed to the uncertainty of the group's conclusion because they saw their decisions refuted by data from larger samples due to the growing samples task design. This finding is consistent with that of Ben-Zvi et al. (2012), indicating that engaging students with growing samples task design helps them move away from complete certainty to uncertainty about their inferences. Players related the aspect of instructional design with other sociocultural aspects, such as school life norms or purpose and utility, in order to attribute commitments and entitlements to themselves or other speakers during the GoGAR.

Dismissive connections between the reasoning aspects were also identified in the group's discussion. The characterization of available frequencies based on mathematical norms was considered a forbidden move in the group's discourse due to the aspect of instructional design. The forbiddance of this move seemed important because it helped the participating students to give up on the frequencies of individual cases and notice patterns in the data. This dismissive relation also reminds us of the role of task design in inferential activities and of the careful selection of problem context that may help students move towards more sophisticated forms of reasoning (Makar & Ben-Zvi, 2011). Some other dismissive relations were also traced in the students' discussion. The dismissive relation of certainty with variability was anticipated because variability is closely connected with uncertainty. In our study, norms of group discussion appeared to be more persuasive for students to give up certainty about the final decision than statistical features such as variability or sociocultural features relating to instructional design. This relation of aspects may suggest that adherence to group discussion norms was a prevailing rule in the group's discourse at that time, likely influenced by a classroom environment that emphasized majority rule in decision-making.

As for the use of background theory in this study, the conceptualization of students' IIR as both social and statistical viewed through the lens of inferentialism seems to highlight the importance of students' previous experiences in reaching conclusions as well as their adherence to implicit rules such as norms. In addition to aspects of reasoning concerning the available data, students in our study drew on their personal experiences and felt the need to consider the implicit rules that prevailed in their classroom while inferring from a given sample. Moreover, by focusing on how students connect concepts to achieve understanding, inferential theory may illuminate the dynamic and interconnected nature of knowledge domains such as statistics. The present research, adopting the theory of inferentialism, aimed to enrich the findings of studies examining social issues related to statistical education by highlighting the normative factors that may come into play as young elementary school students engage in inferential classroom activities.

It is crucial to acknowledge that the findings of the study presented here concern a particular group of students with specific communication skills, academic records, and classroom discursive practices and hence cannot be generalized. However, the study highlights noteworthy features of the connection between students' commitments and the mode of participation in a group while engaging in inferential activities. Specifically, both the expression of commitments and their evaluation seem to be linked to how each student participates in the group discussion and the corresponding dynamics that develop within the group. For instance, a group like the one presented in this study in which students engage in deontic scorekeeping while discussing in order to reach a conclusion may develop an argumentative culture that might lead to the rejection of commitments that are not useful for the development of IIR such as the adoption of certainty or the use of personal beliefs in drawing conclusions. For groups in which students adopt a different participating pattern and do not invite their peers to defend their commitments, certain commitments might be accepted or excluded without negotiation. It would be valuable for future studies to investigate classroom discussions of varied modes of students' participation using the lens of inferentialism to explore further this participatory aspect of making inferences. We also need studies that will focus on teachers' and students' GoGar while engaging in inferential activities. Their findings would offer new insights regarding the role of the teacher in expanding or modifying students' space of reasons about statistical concepts and procedures.

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